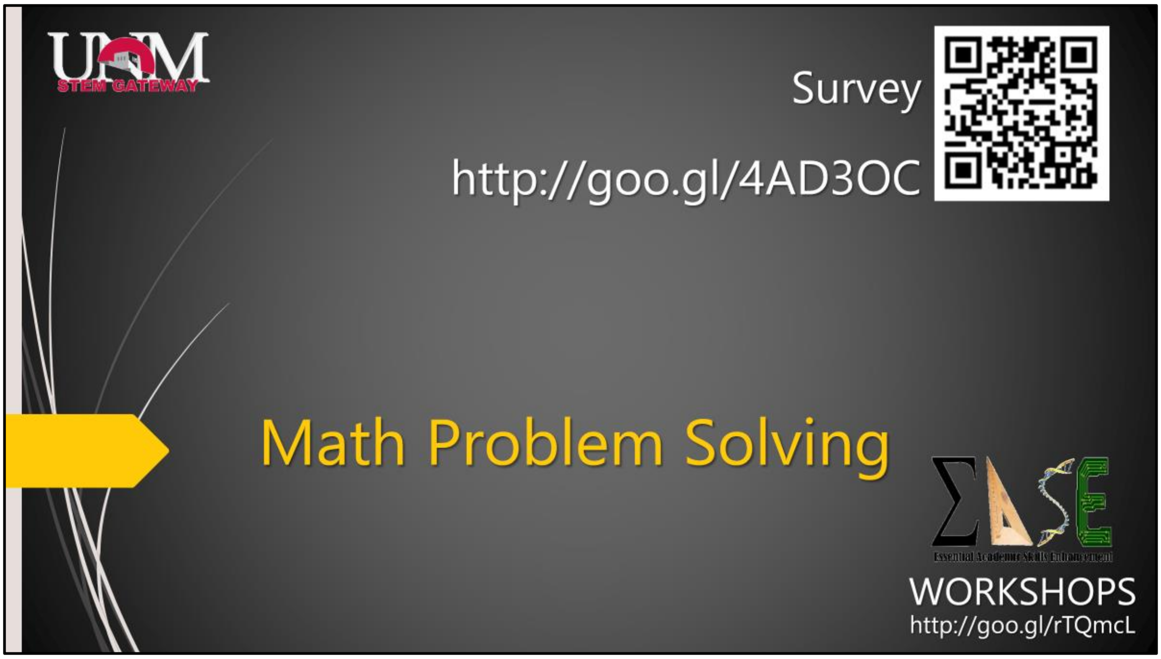


First things first. Welcome to the Math Problem Solving workshop, the second workshop in this series, and part of the EASE workshop series, organized by the STEM Gateway program and Engineering Student Services.

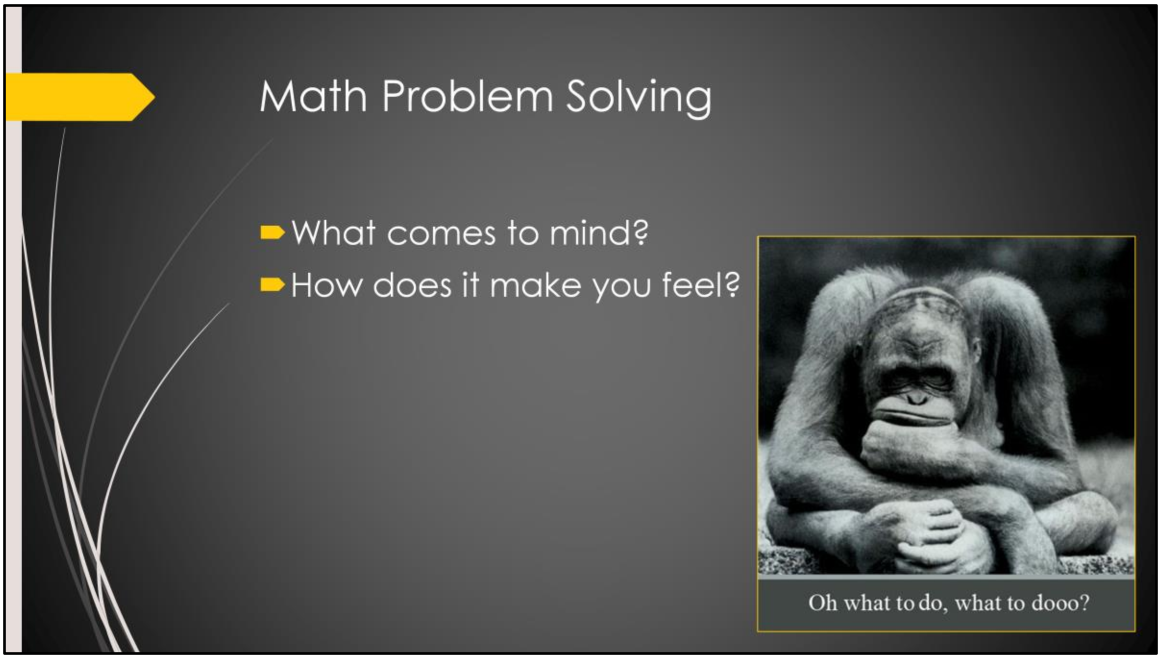
At the end of the workshop, you'll have to complete a survey for us. If you want to do the survey electronically (which is much appreciated), here are the QR code and short URL. If you'd prefer to do a hard copy version, I'll pass them out at the end of the presentation. Before you are free to go, please show me the electronic confirmation, or hand in the paper evaluation, as you leave.

I also want to point out that this Power Point will also be available on the this site through the EASE Workshop series page, so you can always refer back to it at a later time if necessary.

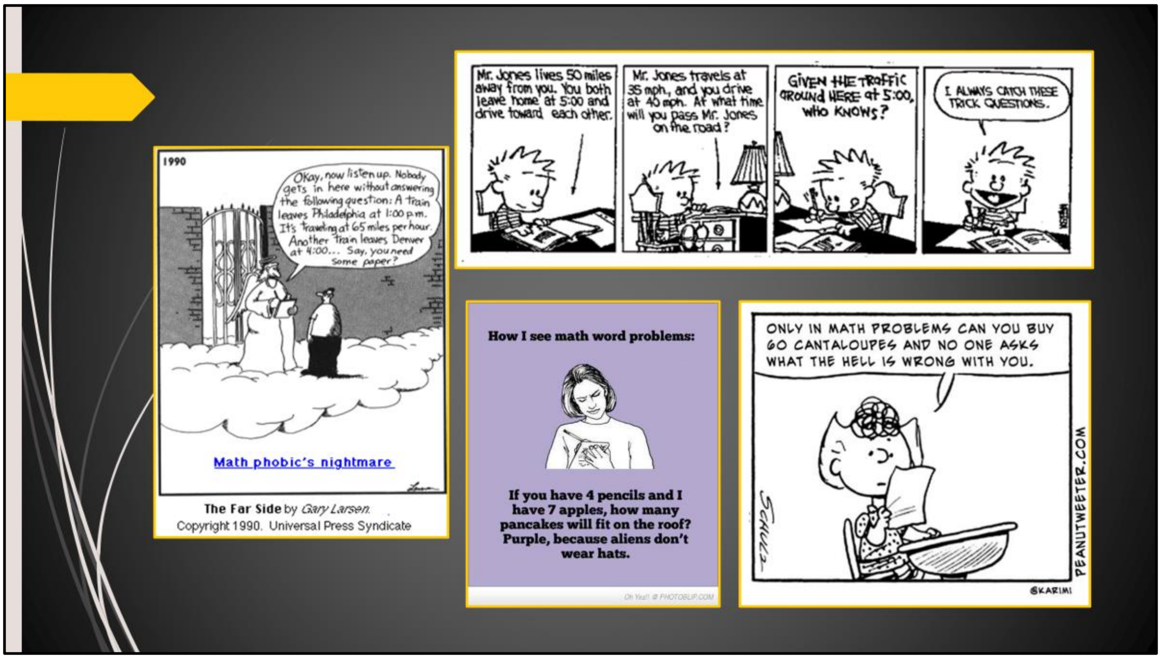
Remember – Tutors are available

- Peer Learning Facilitators (PLFs)
- CAPS

Before we get too far into the Math Problem Solving, I want to point out two tutoring options available to you, if you feel you need more help than this single workshop refresher. ESS and CAPS have tutors that are FREE and can help you if you need it.

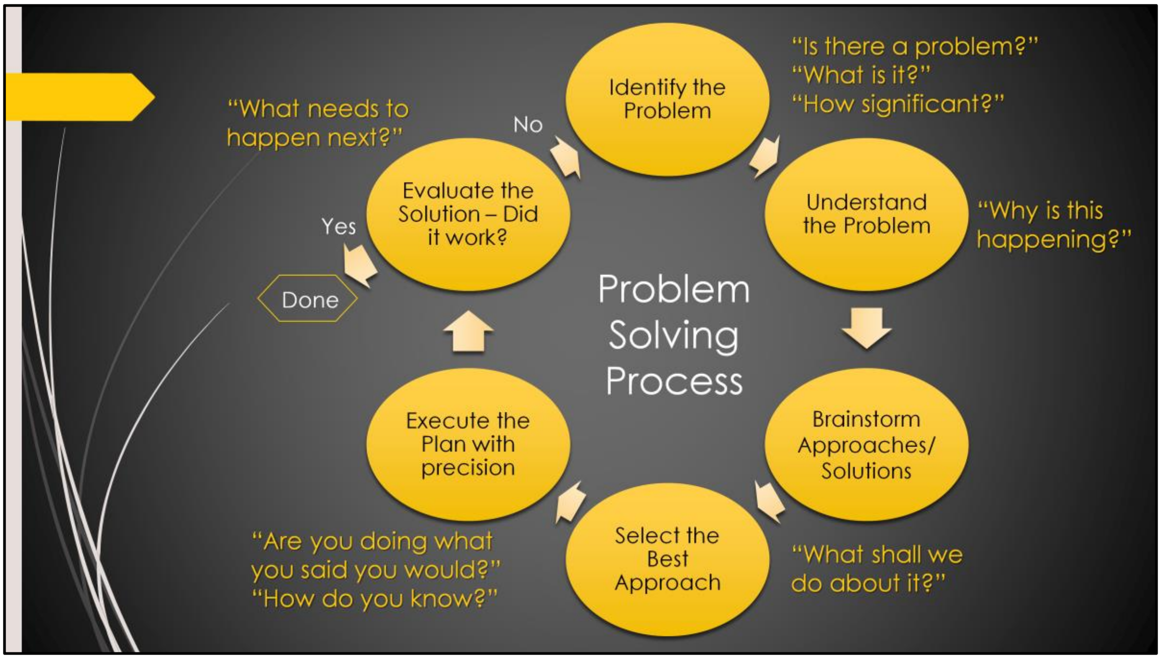


When I say the words “Math Problem Solving”, or in other words, “Word problems”, what comes to mind? How do those phrases make you feel? (discuss)



Perhaps it is something along all of these lines. (Read jokes)

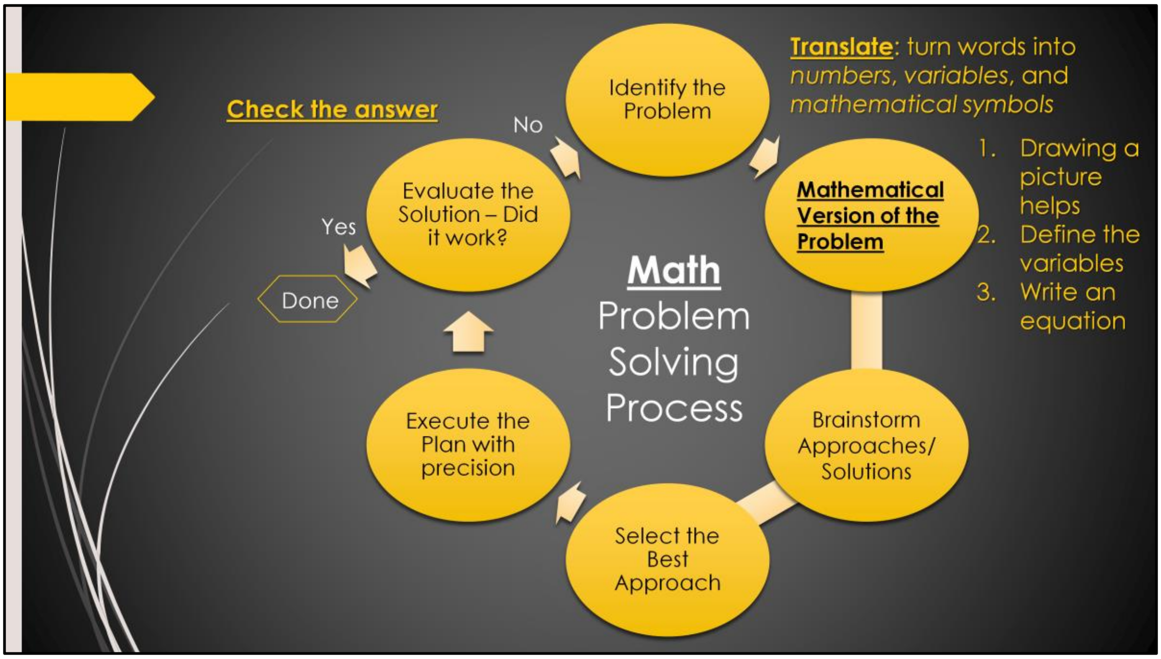
We're here to help with this!



To help eliminate the confusion and thoughts of trickery, using a problem solving process is beneficial.
(*) The first step is to ID the problem, so, is there a problem? What is it? And, how significant is it?
(*) From there, we have to Understand the problem: What is unknown? What are the data? What are the conditions? Can you re-state it in your own words? Is there a picture or diagram that can help you understand this? Is sufficient information given to solve the problem? Be careful of hidden assumptions, data, and conditions.
(*) Next is Brainstorming: find the connection between the data and the unknown, then ask: have you seen this before, perhaps in a different form? Do you know a related problem or theorem that can help? Did in account for all the information provided in the problem? Some approaches are: guess and check, make a list, eliminate possibilities, use direct reasoning, look for patterns, work backwards, be creative.
(*) From all of the potential approaches you came up with, you need to select the best approach: Make sure you discuss your ideas with others when possible.
(*) As you execute the plan: make sure to check each step, can you clearly see each is correct? Can you prove it? Are you doing what you said you would? How do you know?

(*) The last step is to evaluation your work or check the results. Can the solution be derived differently? Can you use this approach on other problems? Did your approach work? If so, then you are done, if not, what is the next step? What is the new problem?

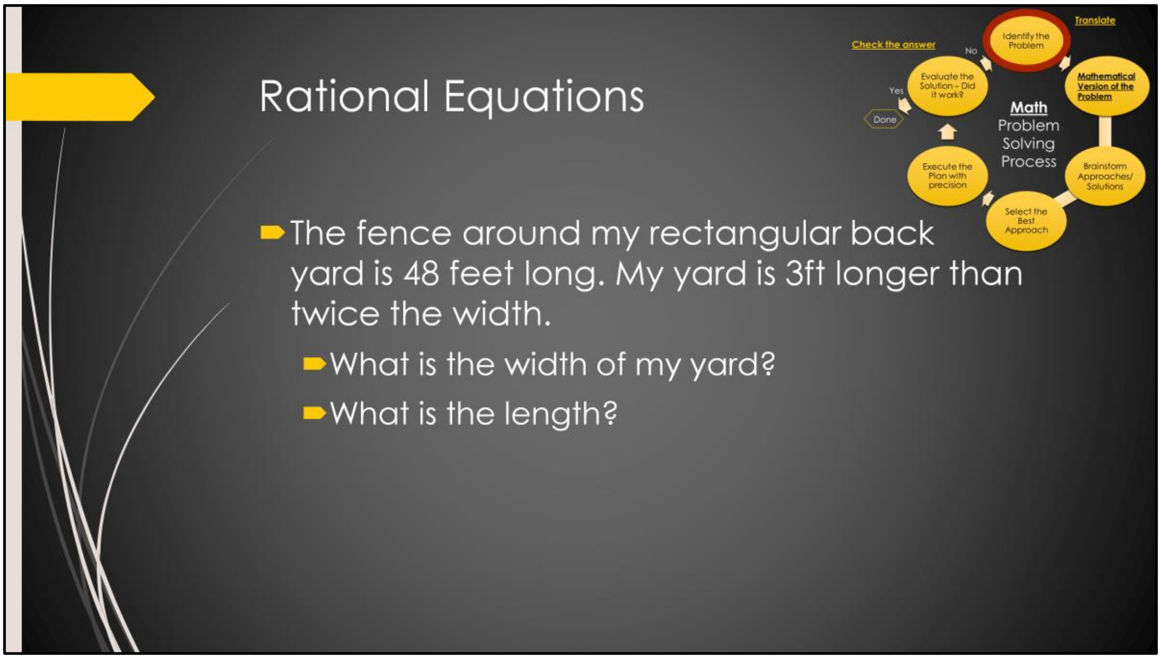
Now, this is all in the abstract, how does it tie in to MATH specifically?



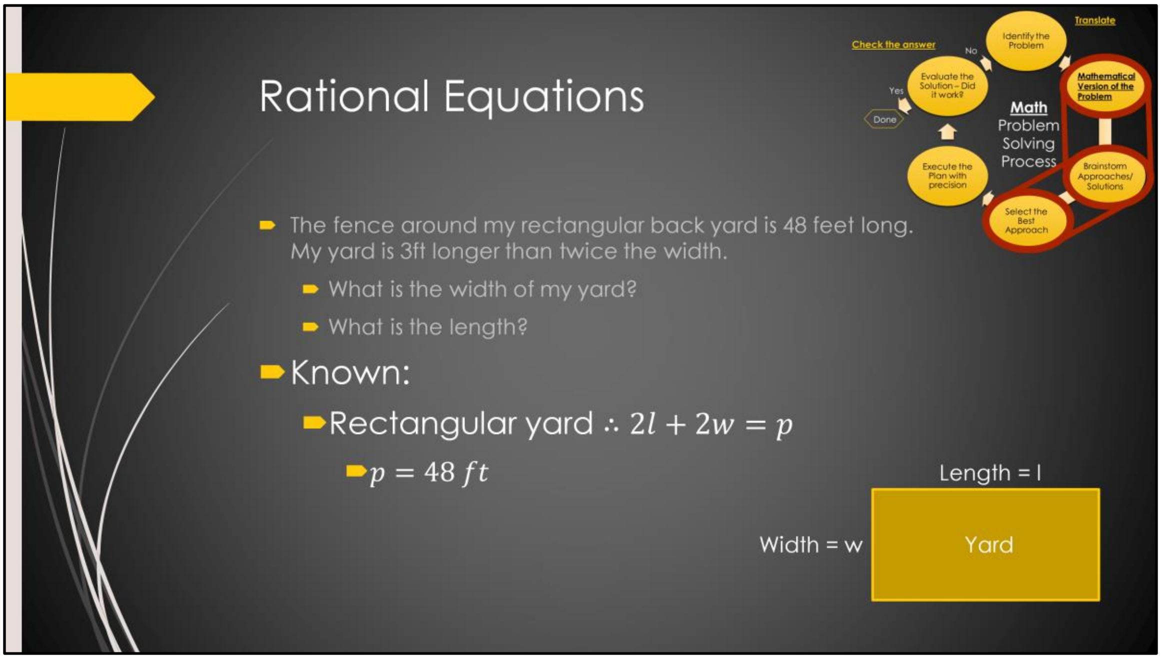
The main connections are between identifying the problem and understanding the problem, we have to translate it into a mathematical version. It helps to draw pictures, define the variables, and write out any possible solutions. The arrows for this step and the next two are turned into connectors, because it is often difficult to differentiate among these steps, and the approach incorporates them all. After selecting an approach, the last two steps are the same. When we are evaluating our work, we make sure to plug our answer back in to the original equation or problem to make sure it works.

Any questions?

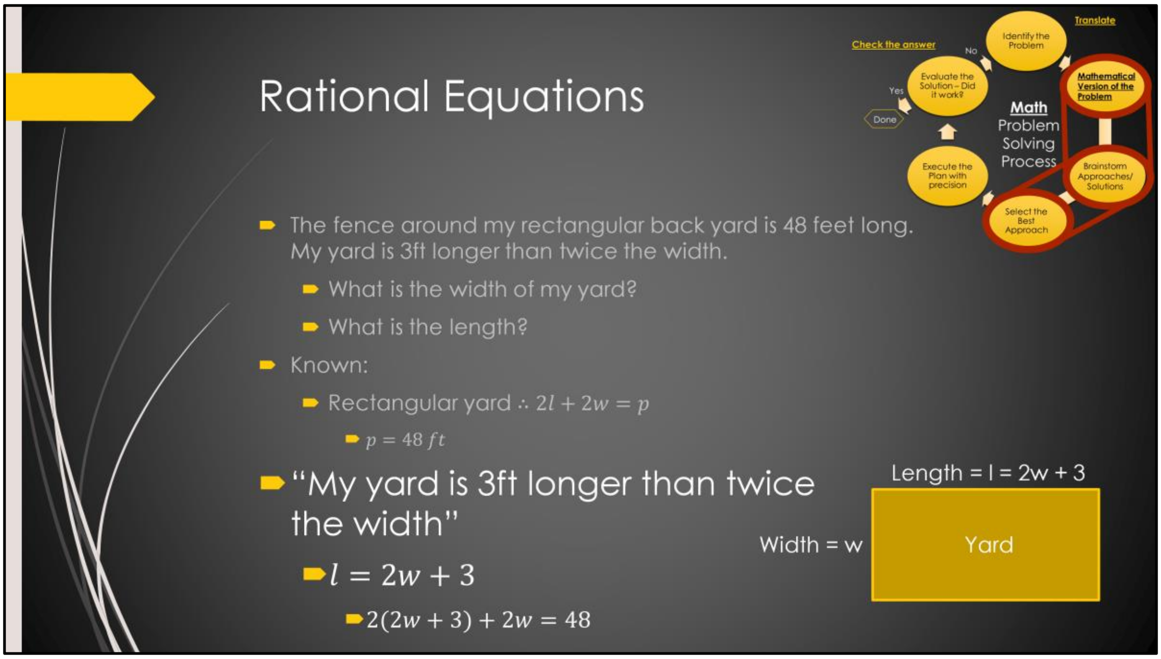
Let's put this in to context.



Here’s is an example that uses a rational equation to solve the problem. For the first step, let’s identify the problem: (Read)

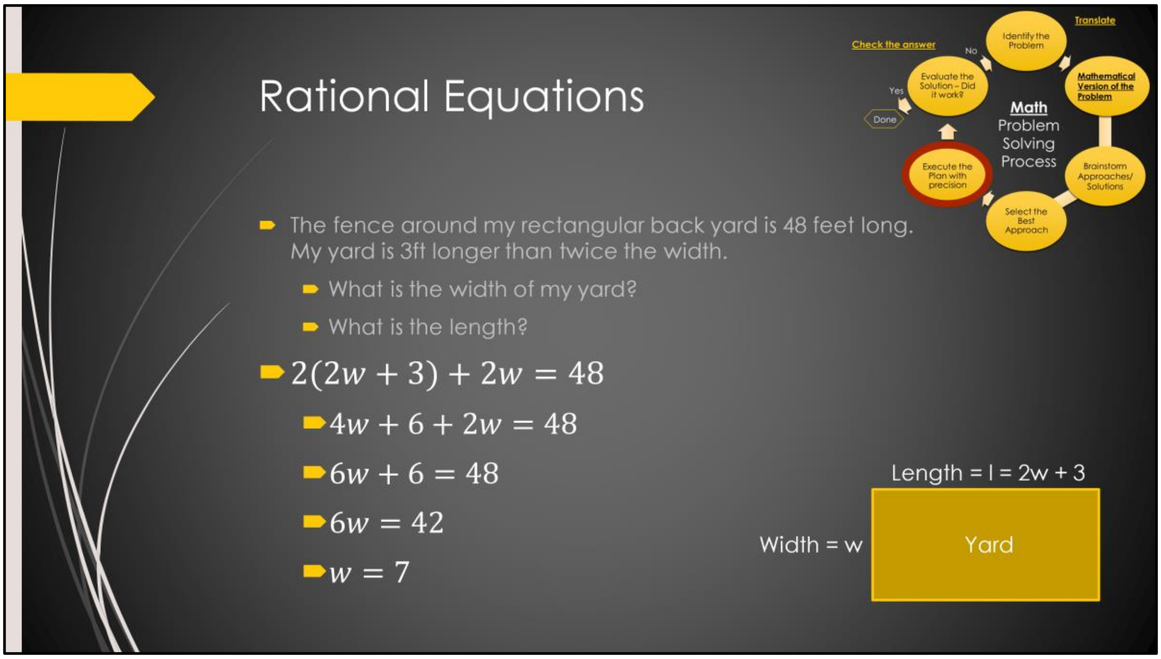


Next is to make the problem into a Mathematical Version of the problem. We need to outline what we know and if there are any equations we can use. Remember, drawing a graphic can help.
(*) Based on the problem, we know that this is a rectangular yard, which has a perimeter of twice the length + twice the width, and we know that the perimeter is 48 feet.
(*) Here is an image that goes with that. This will be helpful in the next step.

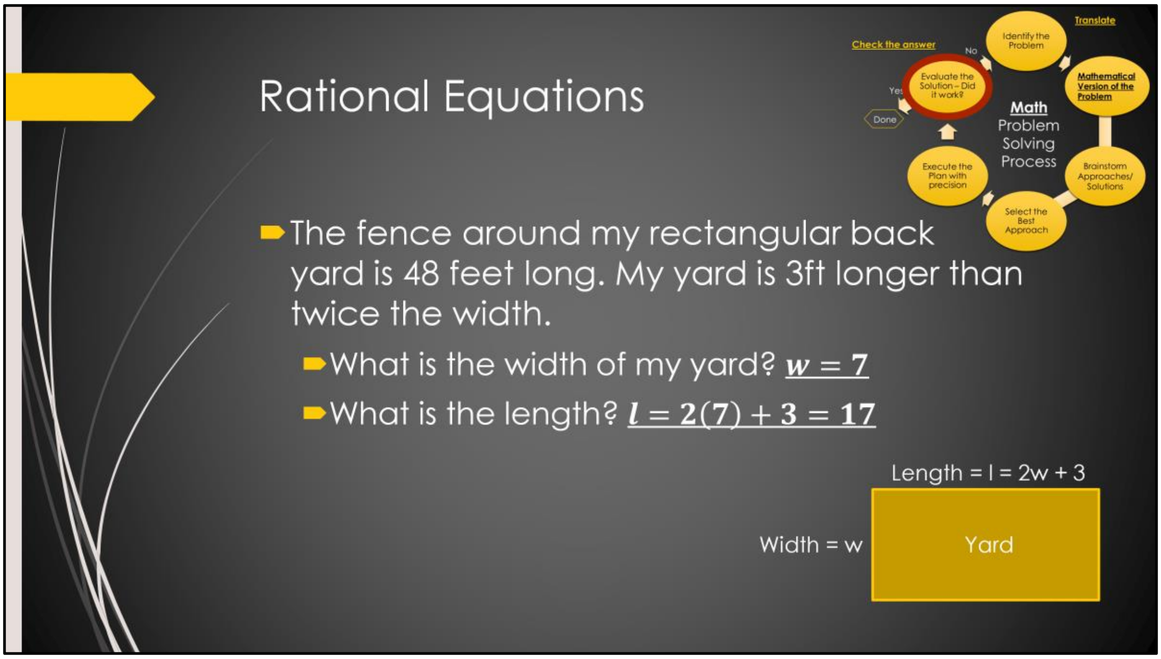


Now as a combination of Brainstorming and selecting the best approach, we have to expand our generic equation with some of the additional information provided in the original problem.

(*) You know that your yard is 3 ft longer than twice the width, that means, the length is 2w + 3, which we can plug in to the original relationship we determined. And, we can update our diagram.

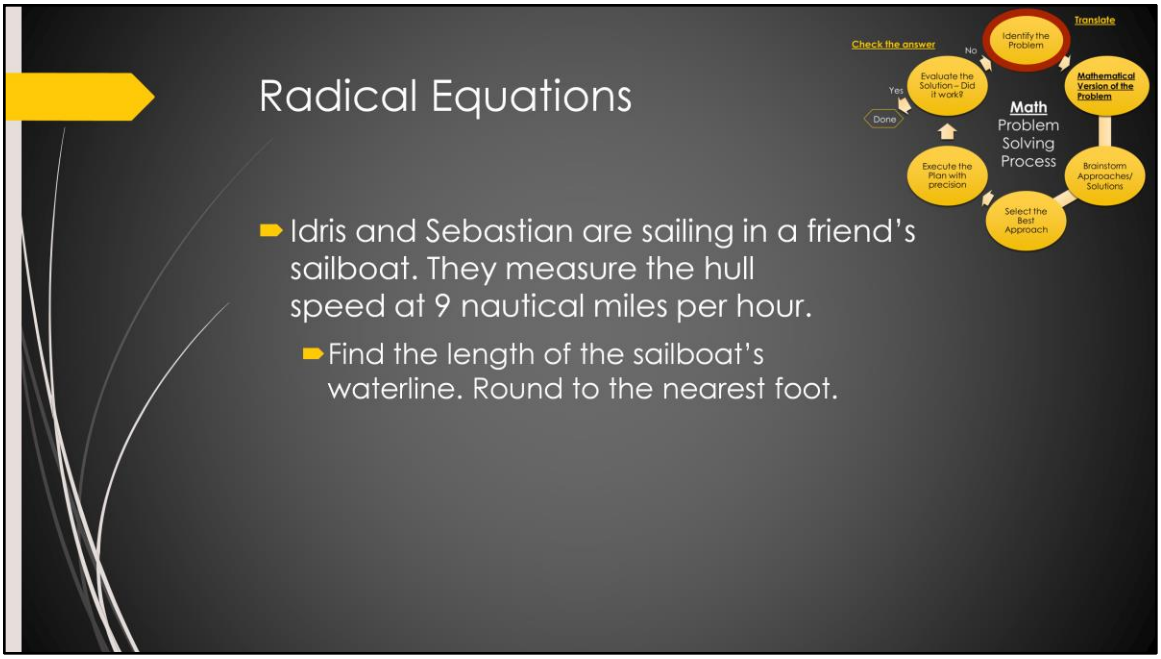


The next step is to evaluate the problem. So, let’s solve the problem using the equation we came up with.

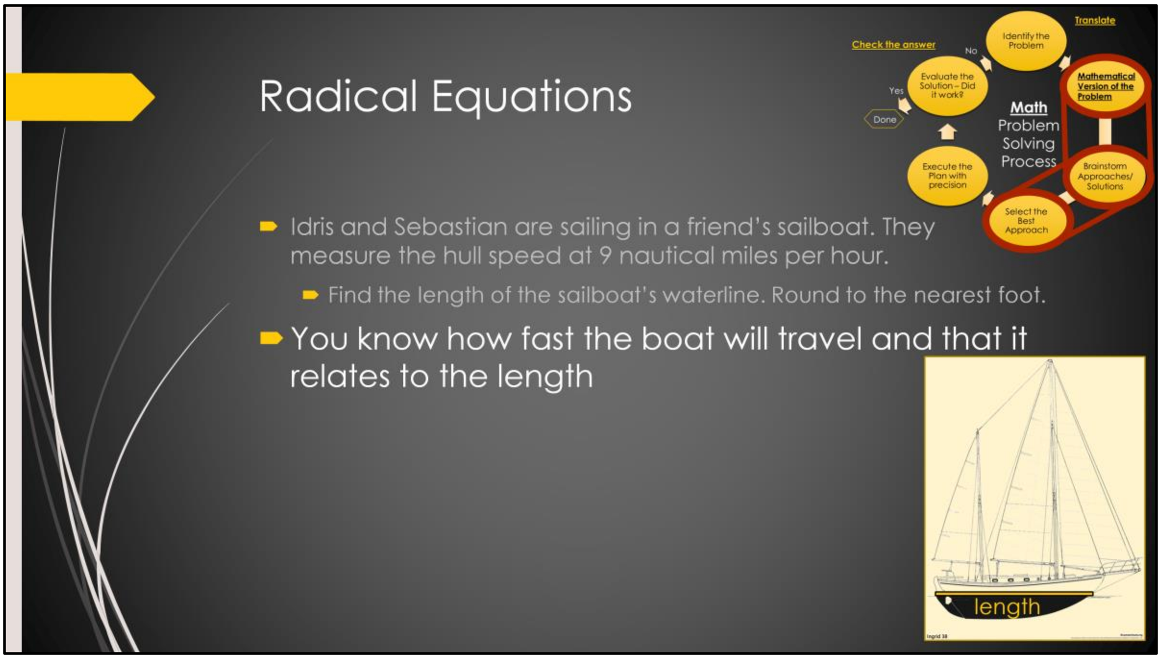


Now we can evaluate our solution by answering the original questions. And we find that the width is 7 and the length is 17.

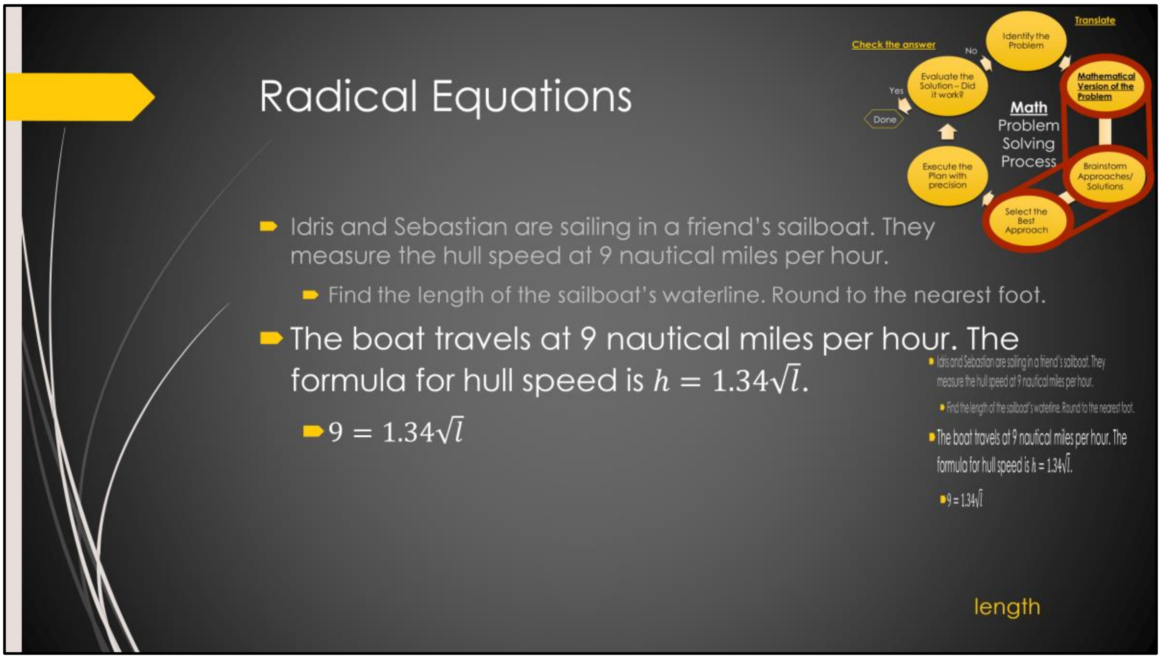
Any questions?



Now let's look at example of a Radical equation word problem. (Read)

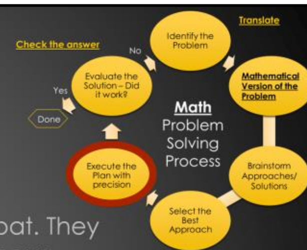


To understand it, or translate into a Mathematical version, we know how fast the boat will travel, and that it relates to length.

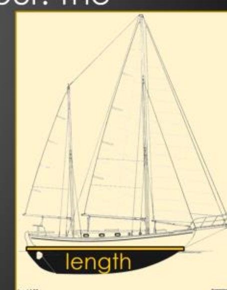


So, we brainstorm what additional information we may need to solve this problem. As a result, we look up the relationship for hull speed, list what is know, and come up with this equation.

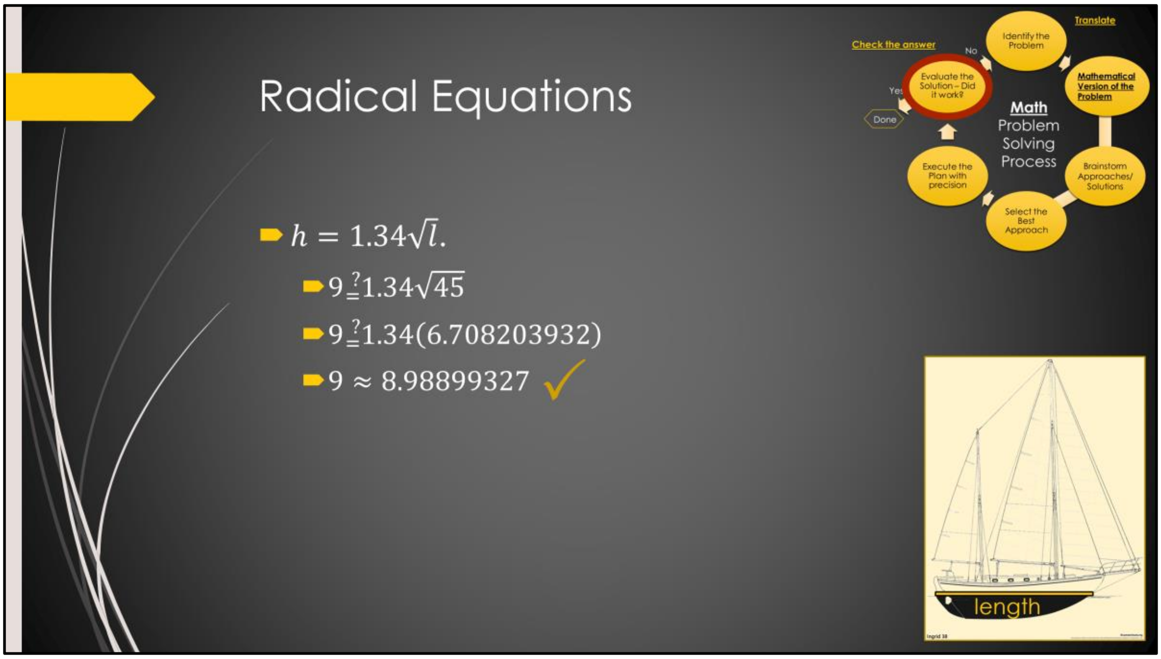
Radical Equations



- Idris and Sebastian are sailing in a friend's sailboat. They measure the hull speed at 9 nautical miles per hour.
  - Find the length of the sailboat's waterline. Round to the nearest foot.
- The boat travels at 9 nautical miles per hour. The formula for hull speed is $h = 1.34\sqrt{l}$.
  - $9 = 1.34\sqrt{l}$
  - $\frac{9}{1.34} = \frac{1.34\sqrt{l}}{1.34}$
  - $6.71641791 = \sqrt{l}$
  - $(6.71641791)^2 = \left(\sqrt{l}\right)^2 = 45.11026954 \approx l$

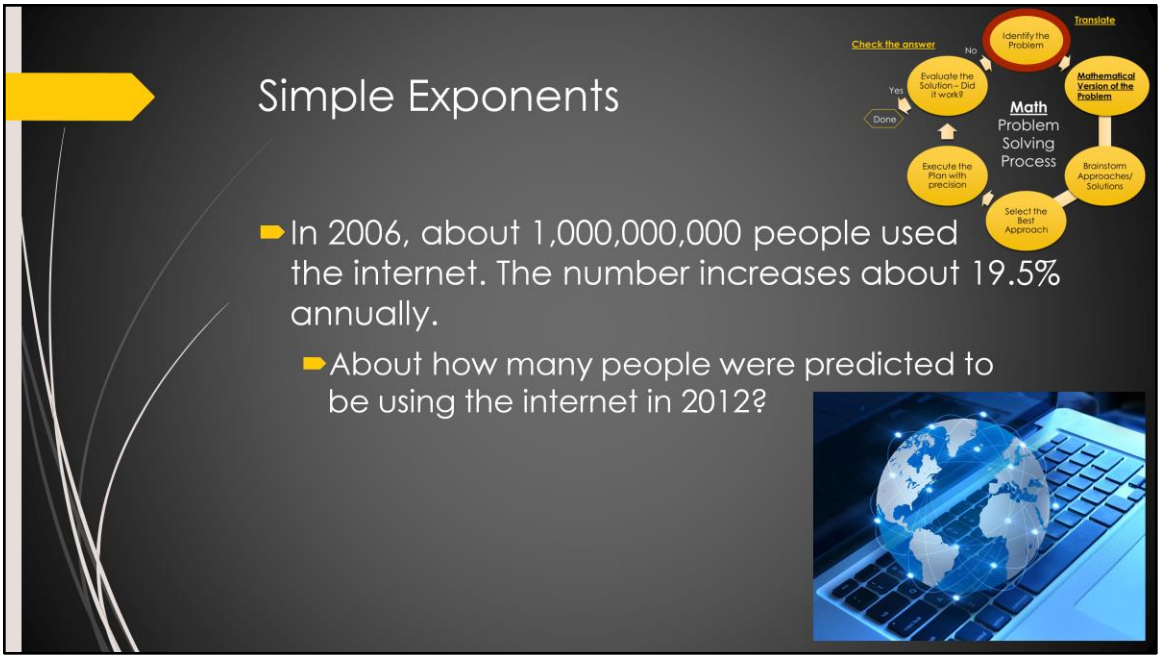


To execute the plan, we solve the equation we came up with.

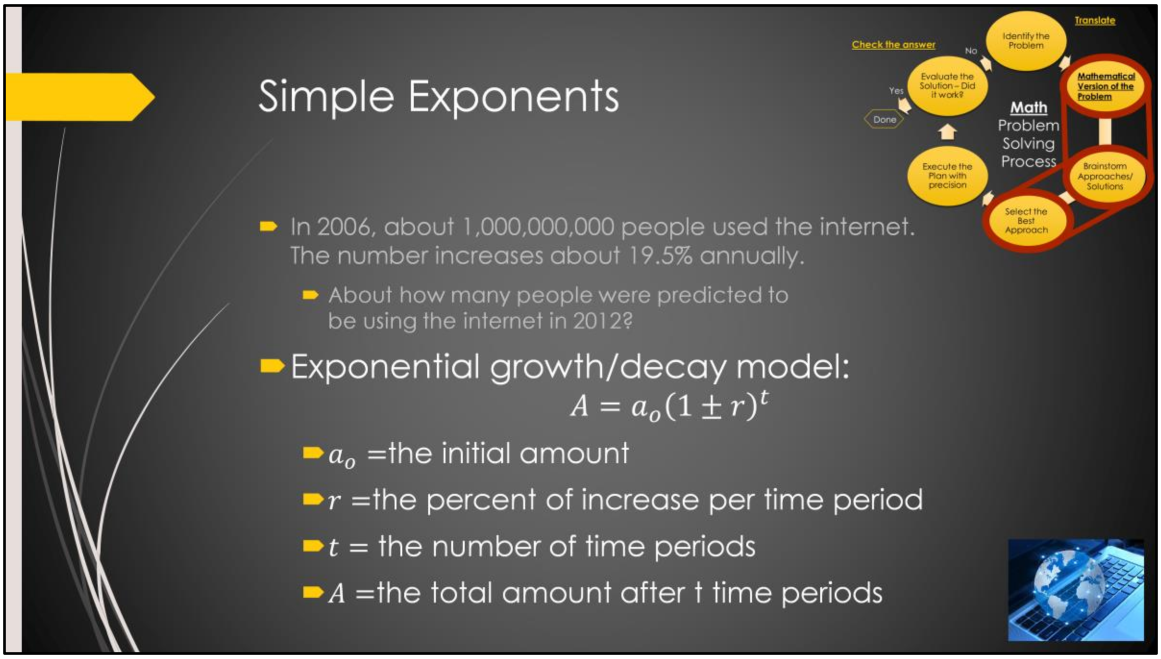


And remember, we have to check our answer, especially when radicals are involved. And yes, low and behold, our approach worked!

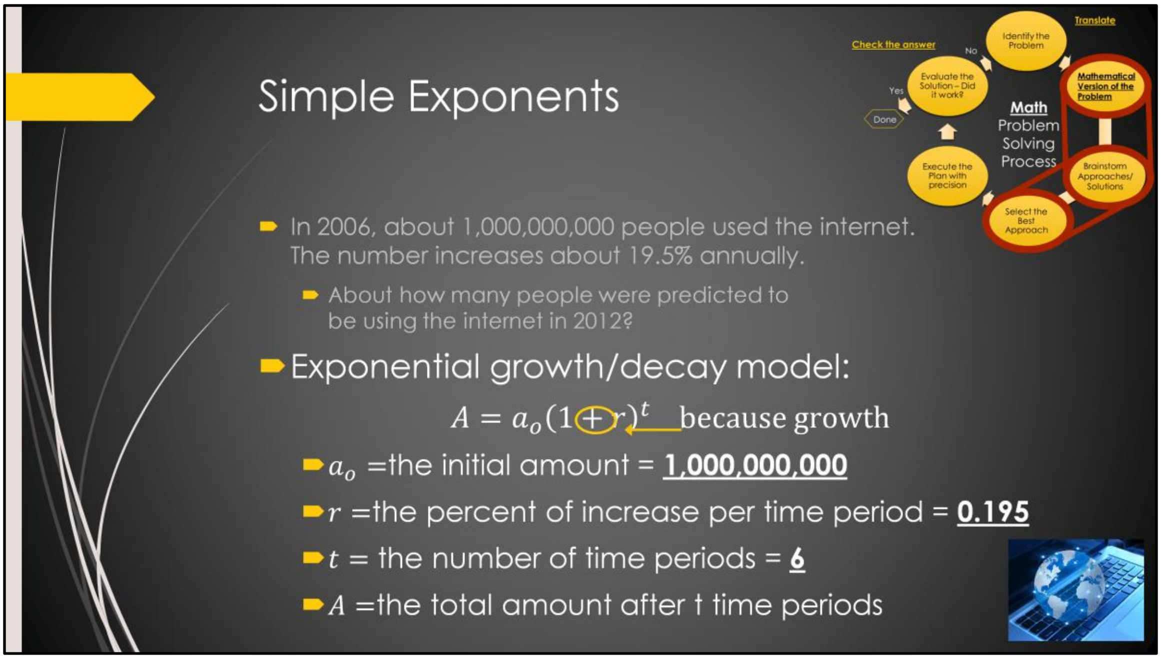
Any questions?



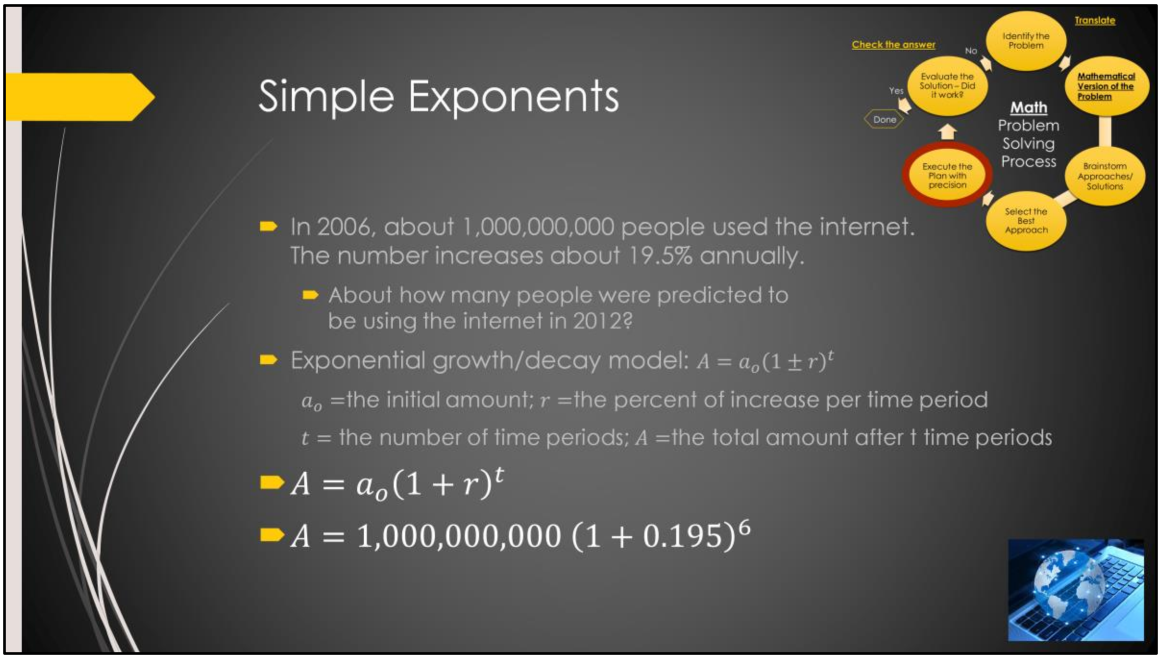
Problems that use simple exponents are usually related to rates of growth or decay in some form. The question here is (read).



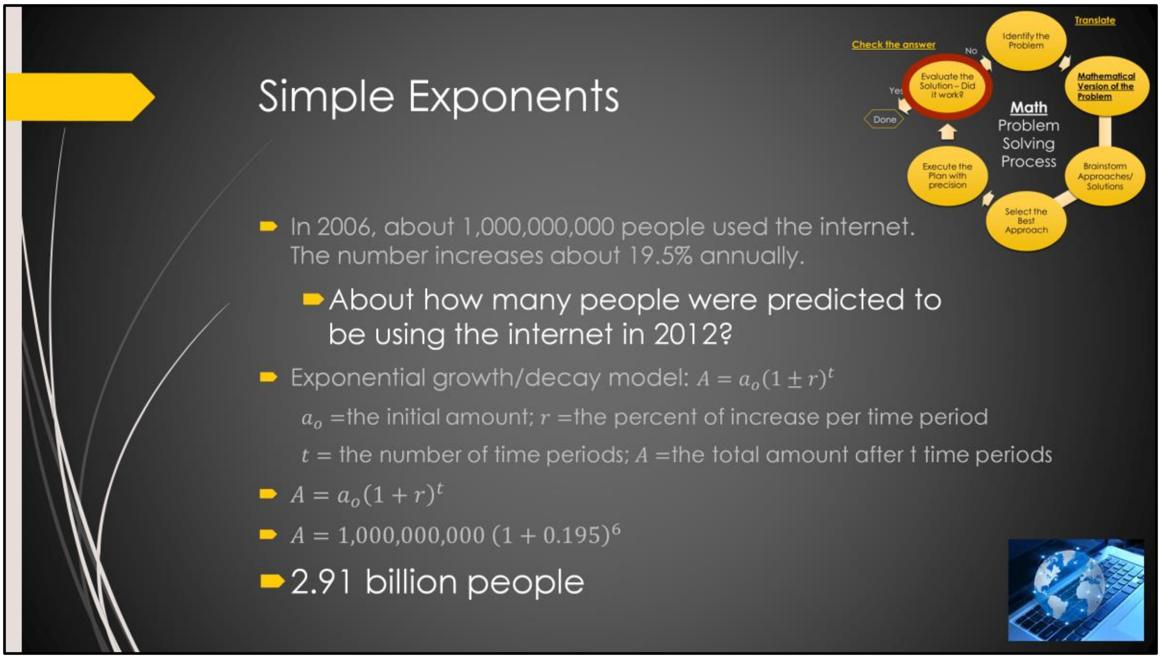
The standard exponential growth/decay model is (read), where we use the + for growth and the – for decay. Our variables are (read). There are other various forms of this equation.



Now that we have the basic equation, we can identify which variables are given in the problem and which form of the equation we will use.

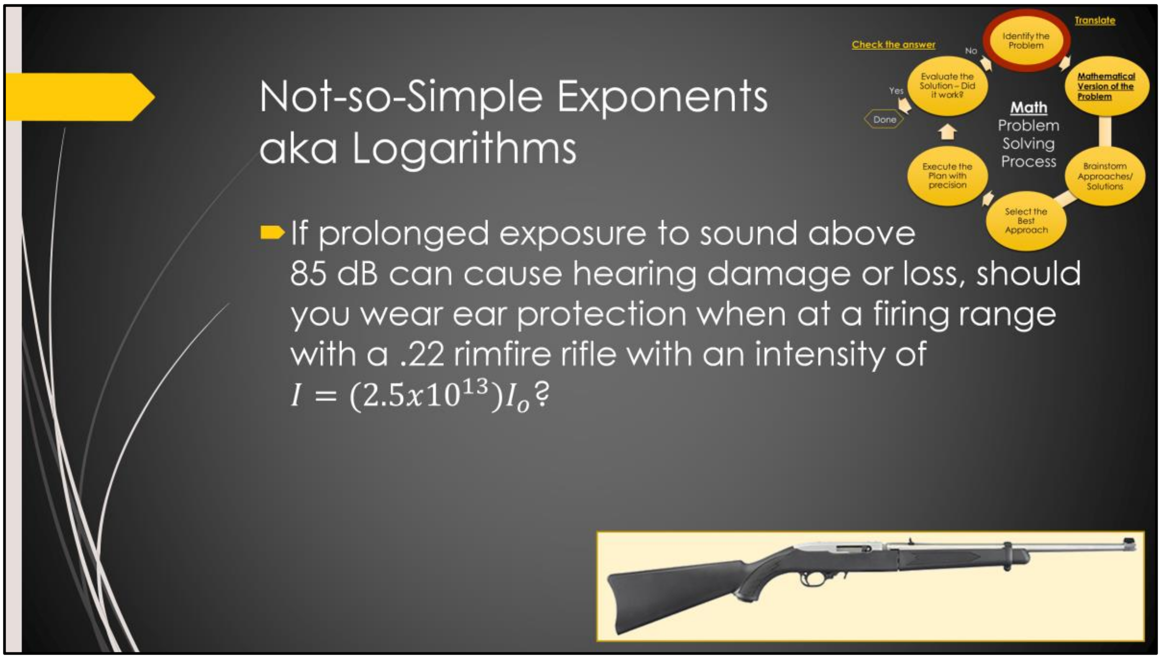


We then plug those values in to the equation



And get an answer of 2.91 billion people were predicted to be using the internet back in 2012.

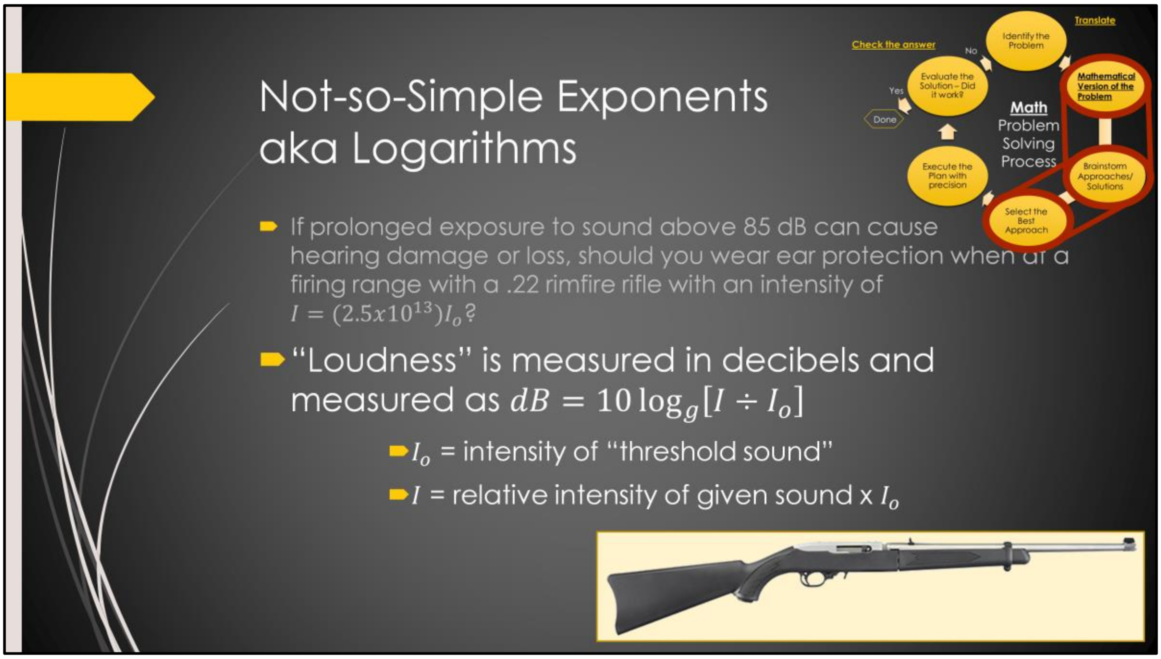
Any questions?



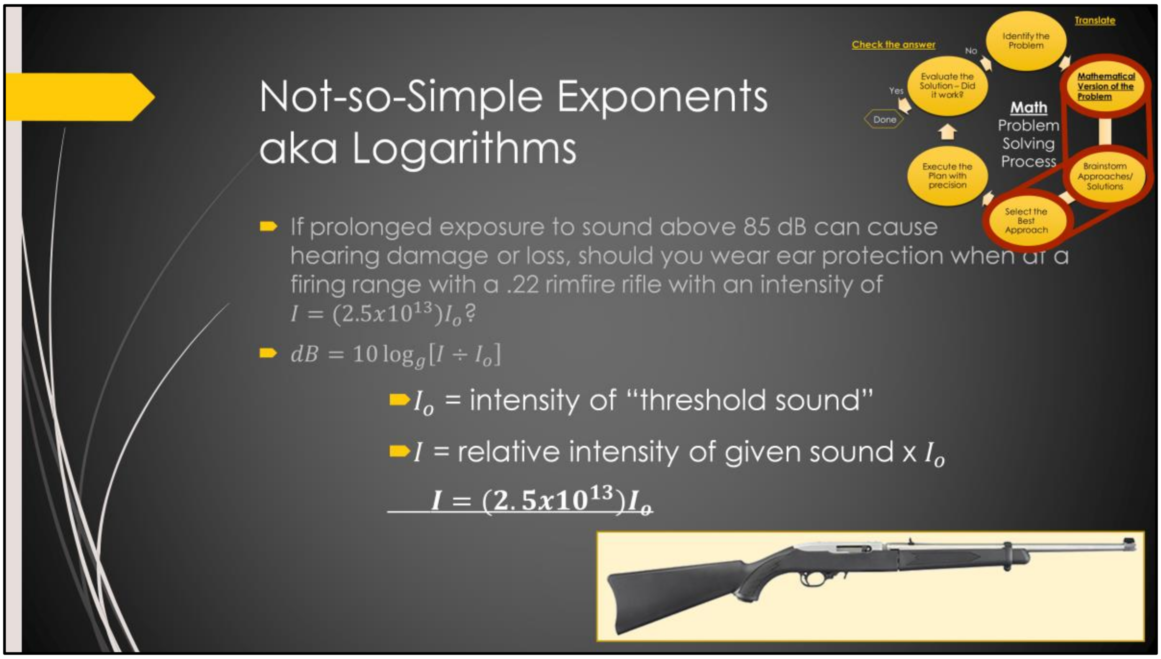
The nice thing about word problems that involve logarithms, is they are often easier to solve than your standard log based problem.

A log-based problem may not be easily identifiable as one in the question itself, unless the equation is given.

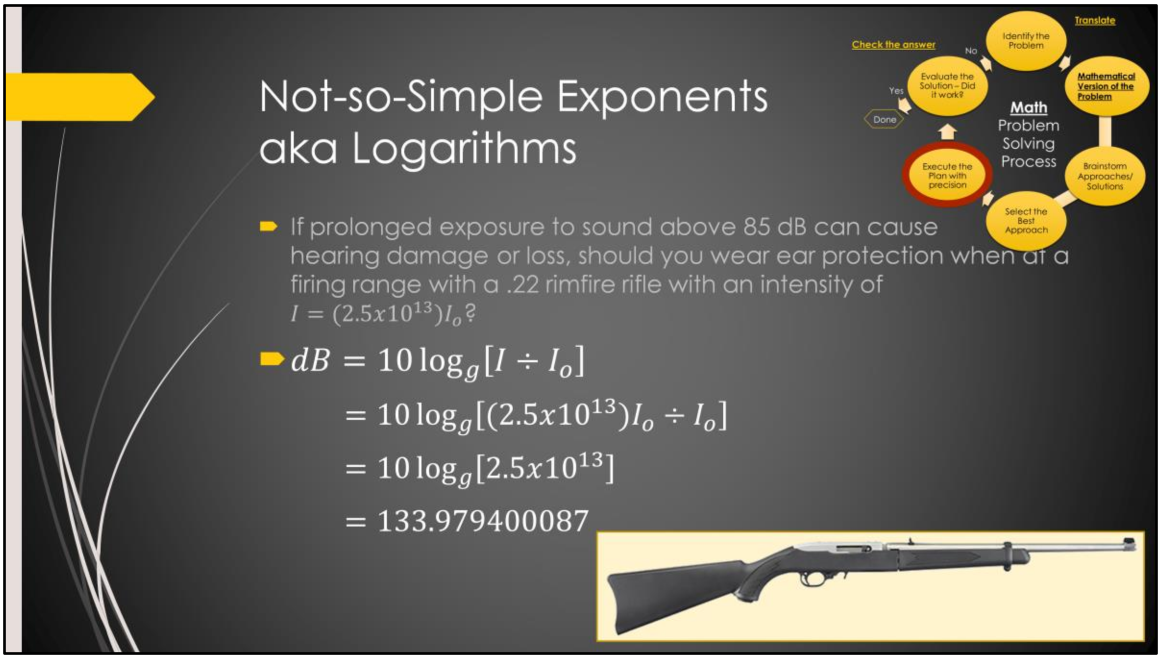
For this question (read), it is like before, the logarithm only becomes apparent when we look for an equation that can help us answer this question.



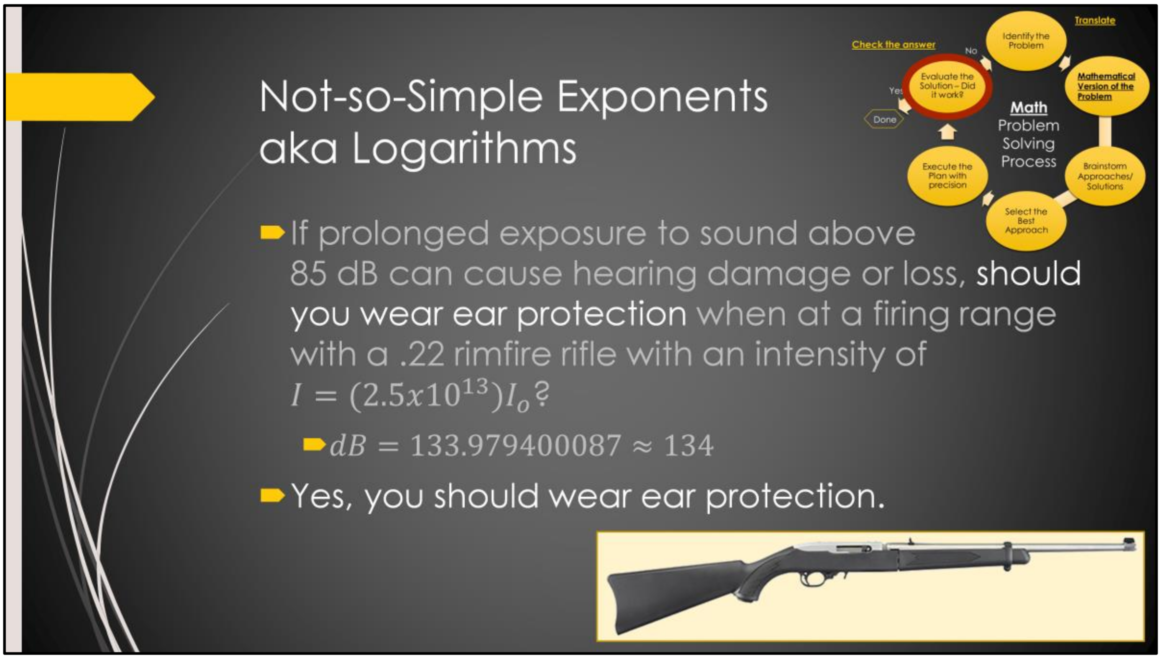
You see in the equation for loudness, that decibels are expressed as (read equation and variable definition).



In the original question, we are given the value of I, so we just need to plug this in to the equation for dB.

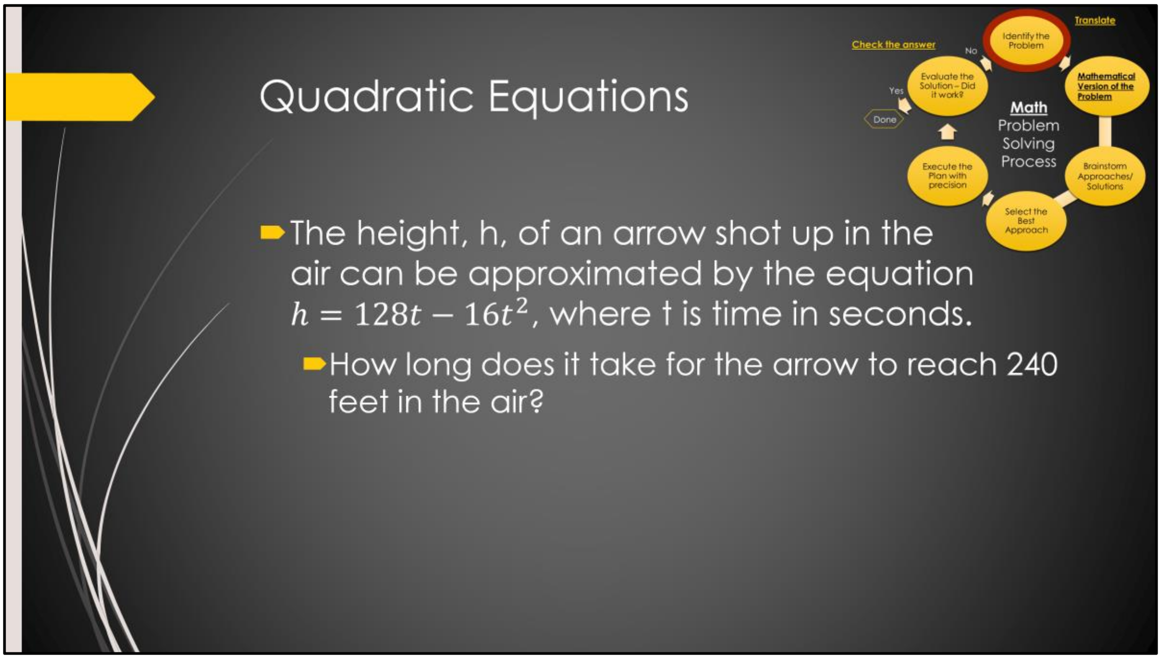


And then solve the equation.



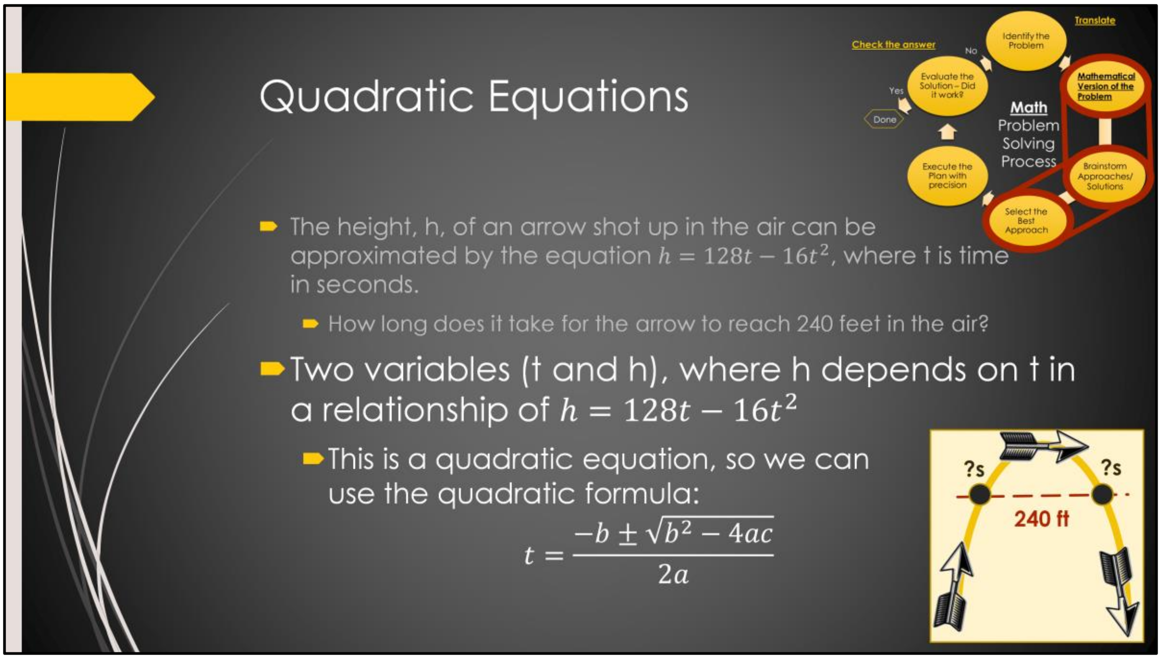
Since our answer is well above 85, yes, we should wear ear protection to avoid long-term hearing loss.

Any questions?

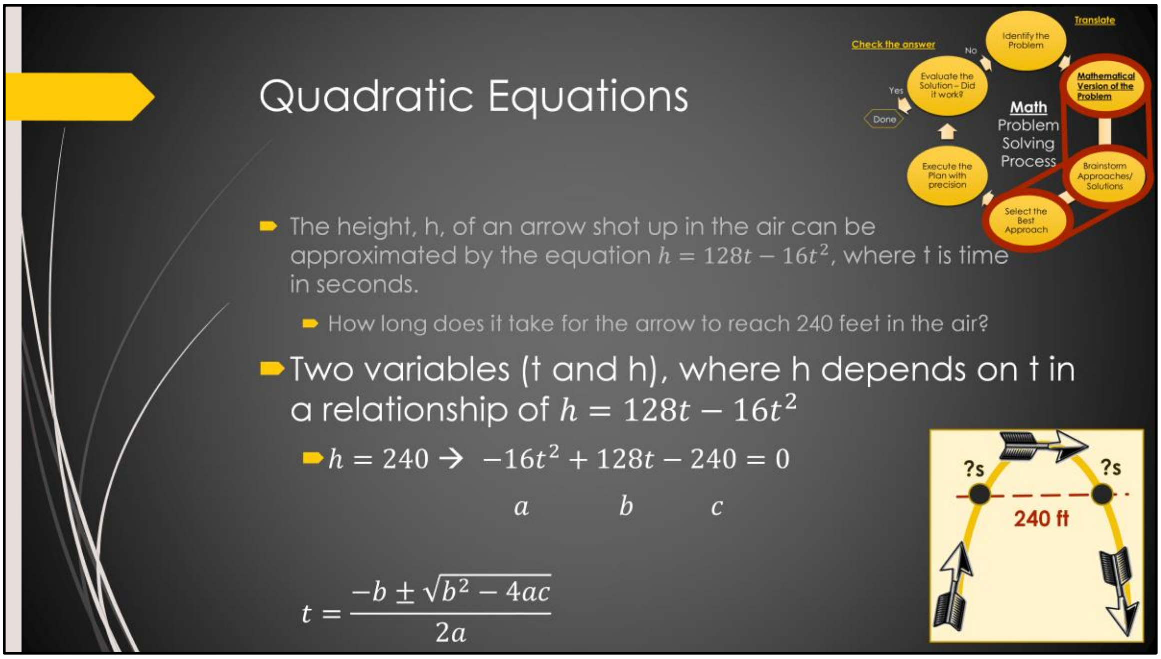


As with logarithms, problems that utilize the a quadratic are usually identified once you have an equation to plug values into, rather than directly stated in the problem.

In this problem (read), the equation is given to us.

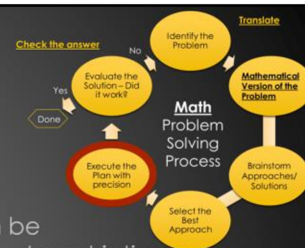


Let's draw what this is asking. The arrow will initially reach 240 ft at one time, but will also be at that height on the way down.
(*) We also know the two variables are t and h, where h depends on t, and the relationship is given as (read), which makes sense if we need 2 possible answers. So, we just have to re-arrange the problem so we can solve it using the quadratic equation.



We then identify which values plug in as a, b and c

Quadratic Equations

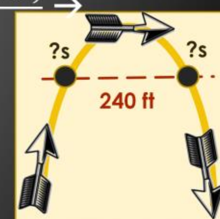


- The height, h, of an arrow shot up in the air can be approximated by the equation $h = 128t - 16t^2$, where t is time in seconds.
  - How long does it take for the arrow to reach 240 feet in the air?
  - $-16t^2 + 128t - 240 = 0$

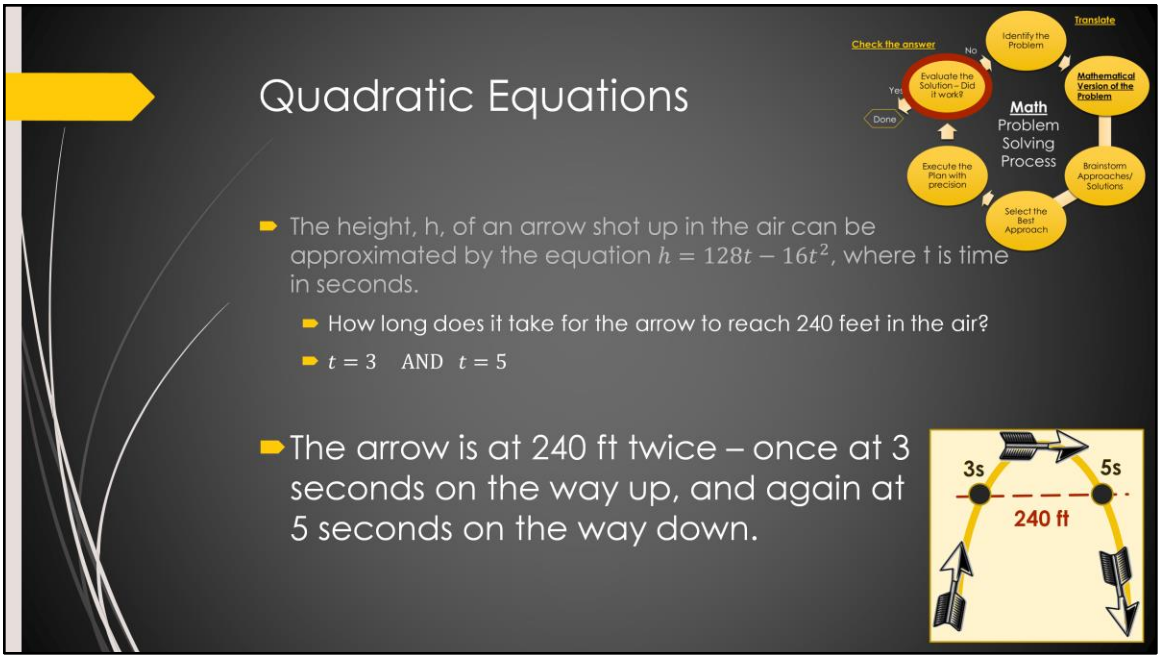
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow t = \frac{-128 \pm \sqrt{128^2 - 4(-16)(-240)}}{2(-16)} \rightarrow$$

$$t = \frac{-128 \pm \sqrt{16384 - 15360}}{-32} \rightarrow t = \frac{-128 \pm 32}{-32} \rightarrow$$

$$t = 3 \quad \text{AND} \quad t = 5$$

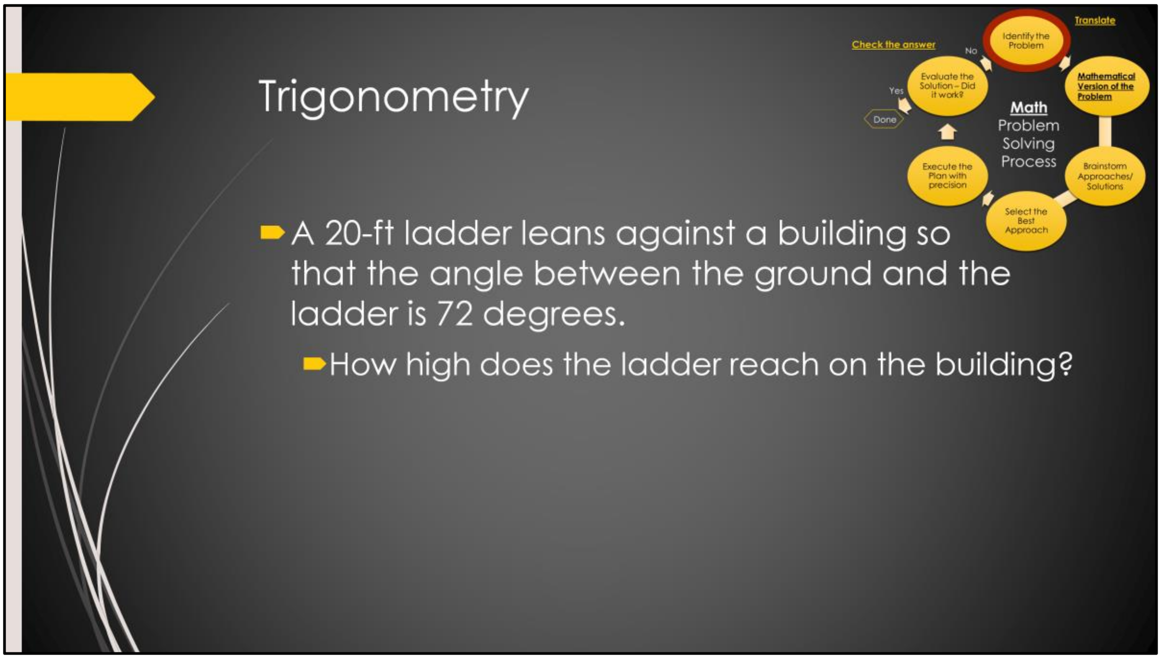


And solve for t.

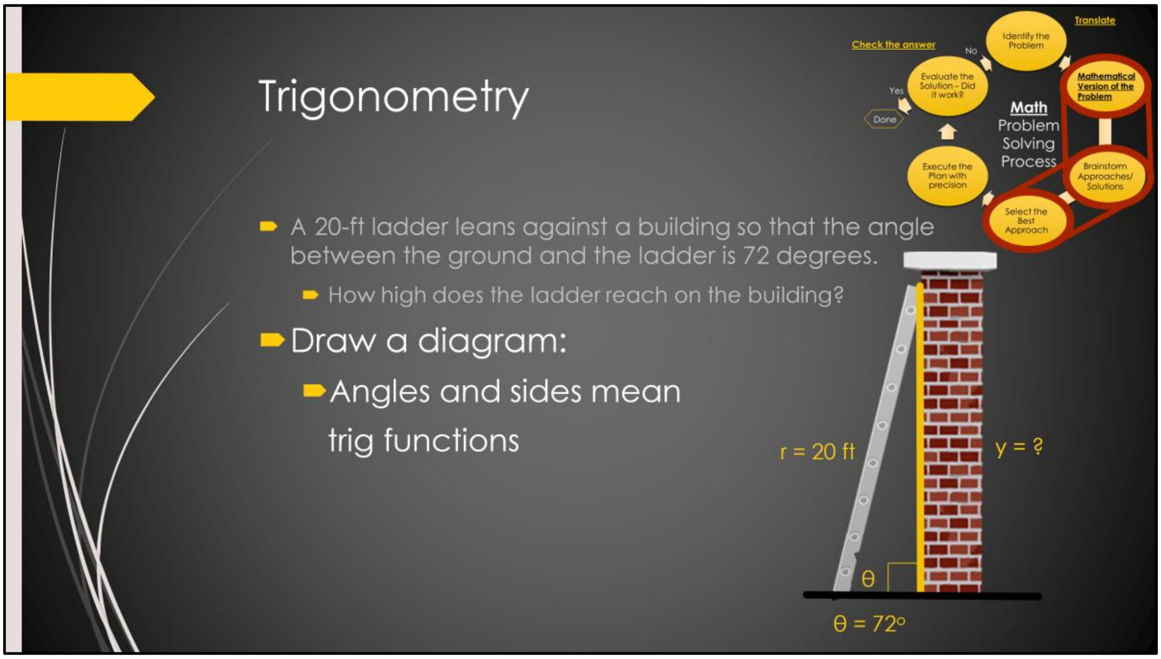


So, t = 3 AND, t=5. That is, the arrow at 240ft twice, once on it’s way up at 3 sends, and again at 5 seconds on the way down. It is thus implied that it is above 240 ft between 3 and 5 seconds.

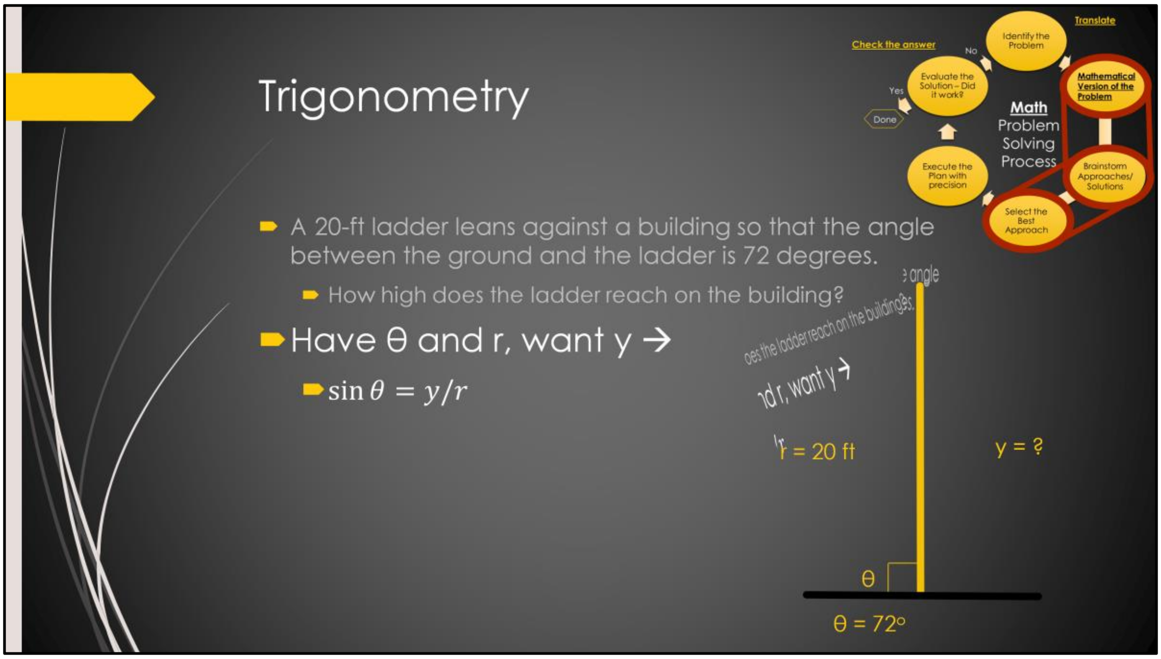
Any questions?



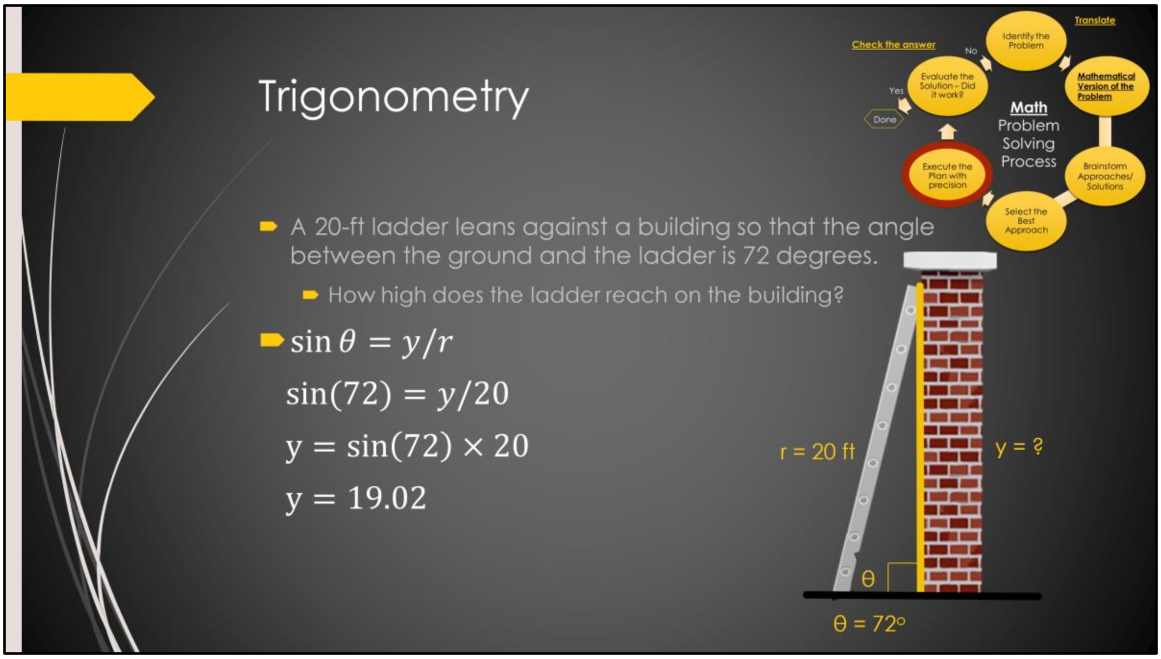
We can identify a problem that uses trigonometric functions because it usually references an "angle." For example, (read question).



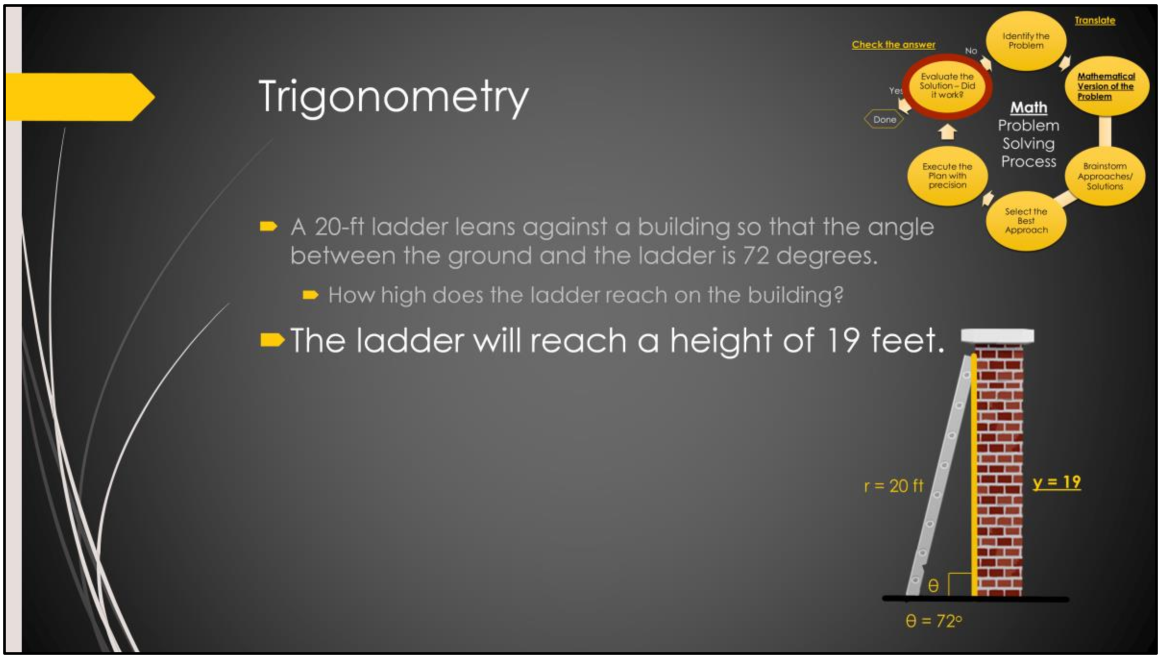
It always helps to draw a diagram when possible. We are looking for the height, and know the hypotenuse of the triangle that the wall and ladder make, with an angle of 72 degrees between the ground and the ladder.



We then identify what we are given and what we want and determine which relationship is best to solve this. In this case, a sin function is best.

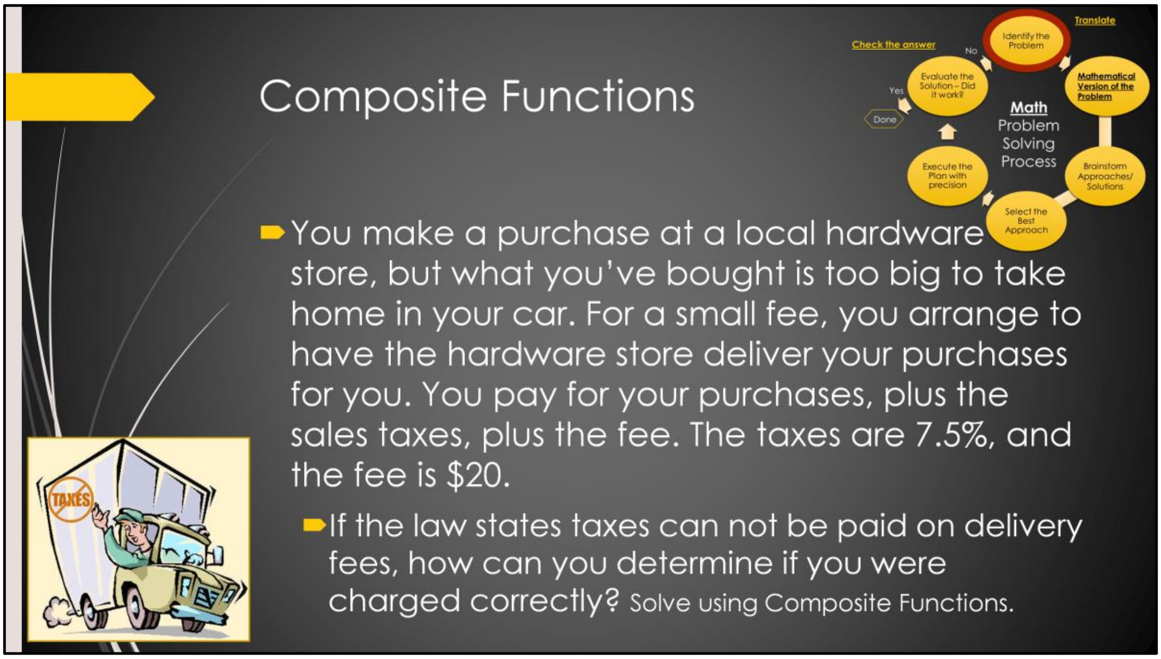


So we plug in the values we already have and solve the equation.



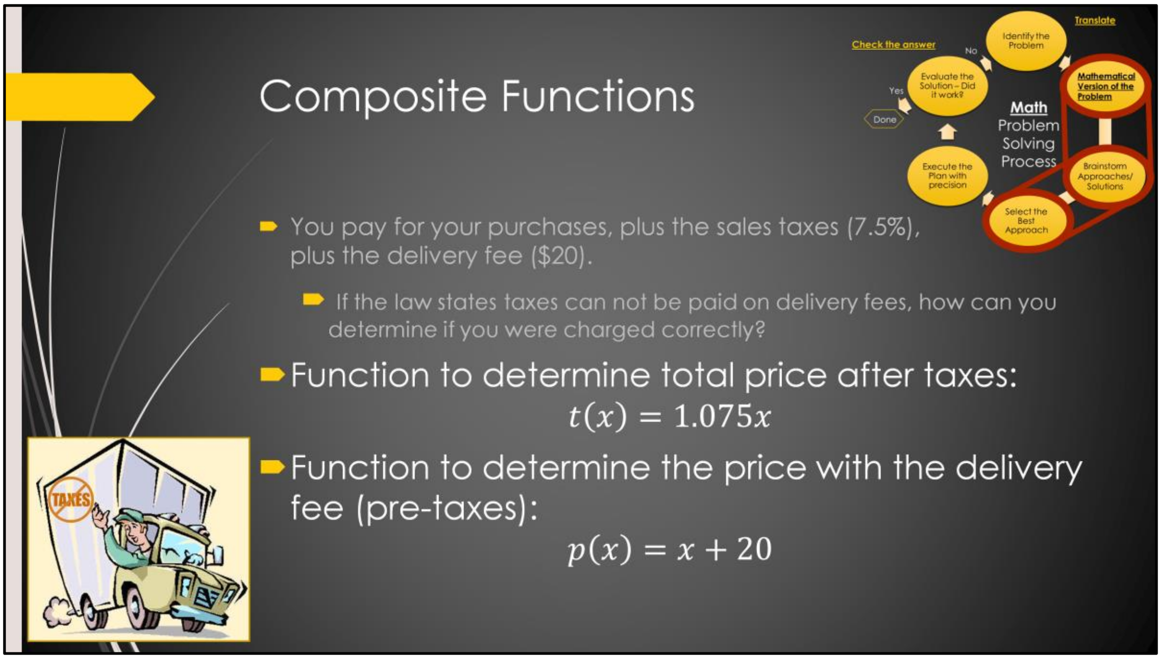
Thus, the ladder will reach a height of 19 feet.

Questions at this time?



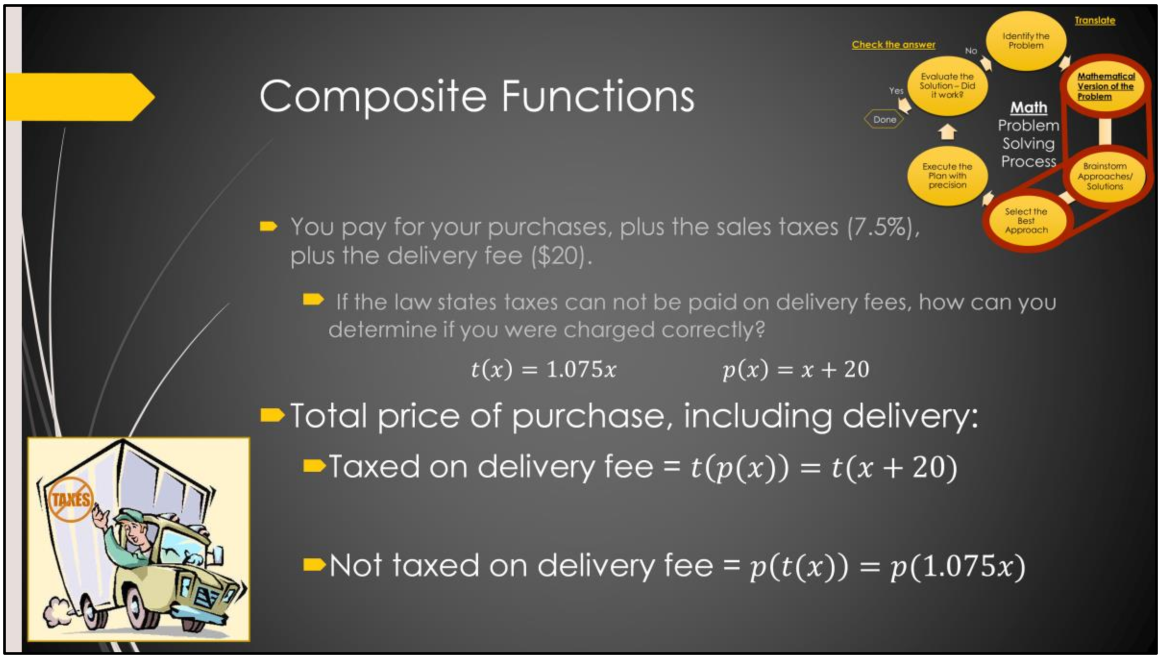
Composite functions become a bit more complex. For example, (read).

Yes, there are multiple ways you could solve this, but let's do it with a composite function for practice.



Since we are told to use a composite function, we know we have to have two separate functions that are in some way related to each other. From the problem itself, we can then determine that the total price after taxes, without the delivery fee is (read). And, the price with the delivery fee (pre-tax) is (read).

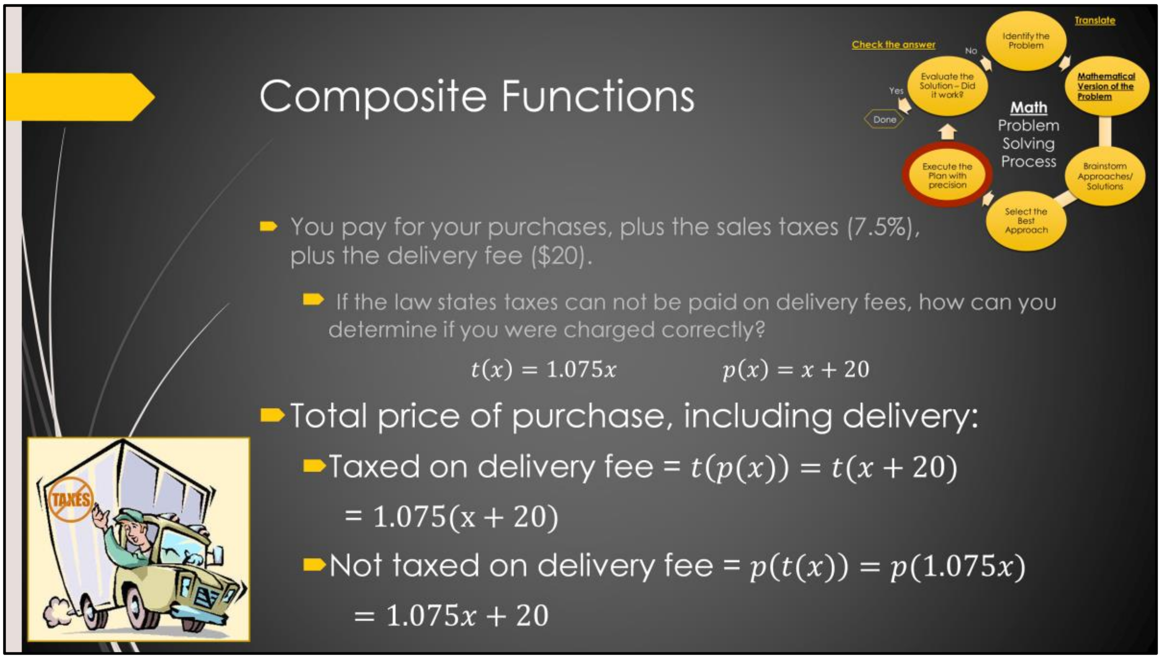
Does that make sense to everyone?



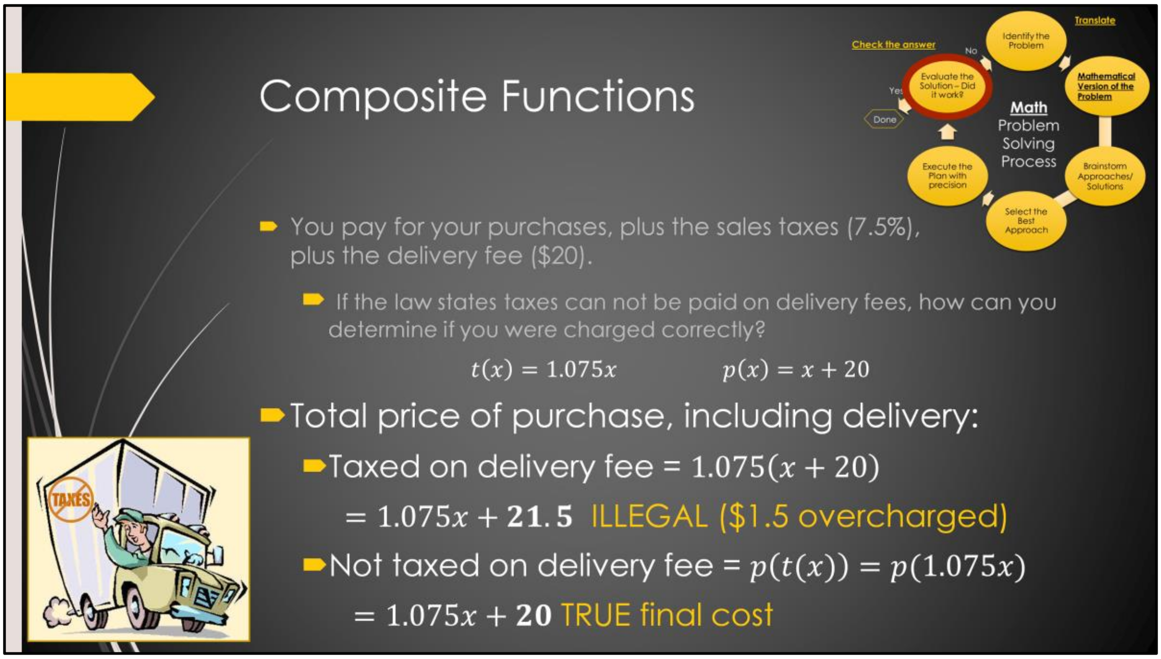
Alright, so next, we have to determine how these functions fit with each other. If we wanted the taxes to be placed on the delivery fee, we know that the total price (with tax) involves the price + delivery fee, so we can write it as (read equation).

Where if we wanted the delivery fee added AFTER taxes are calculated, we first determine the price on the merchandise with tax, then add the delivery fee, through the composite function of (read equation).

Are you following me so far?

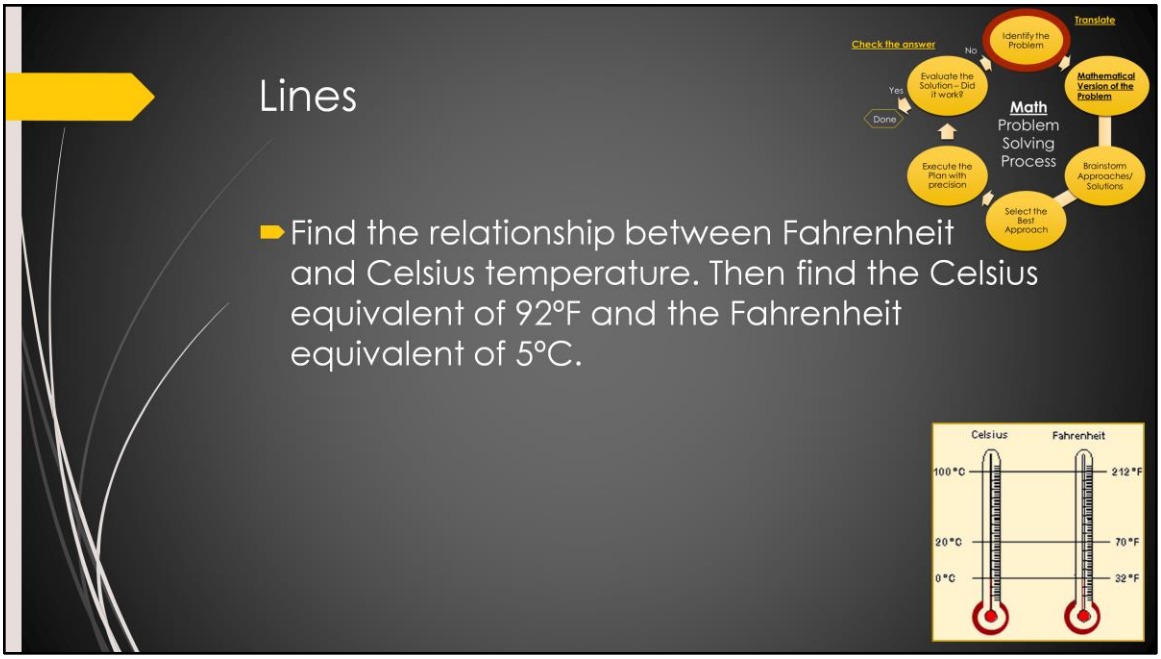


So now, we can plug in the given values for each function to determine the generic equation of each total price. This is what cash registers these days have built in to them.



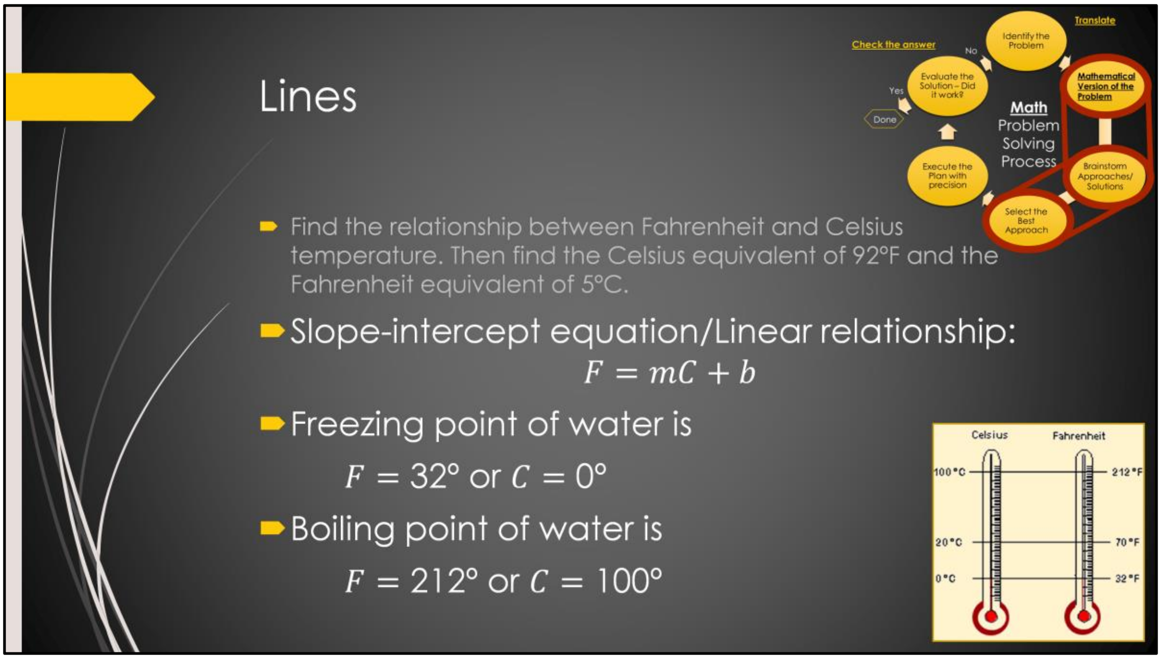
If we were to plug the value of our merchandise into each equation, we'd notice that we would be overcharged by $1.5 if the business partook in illegal practices and charged us taxes on delivery.

Questions on composite functions?

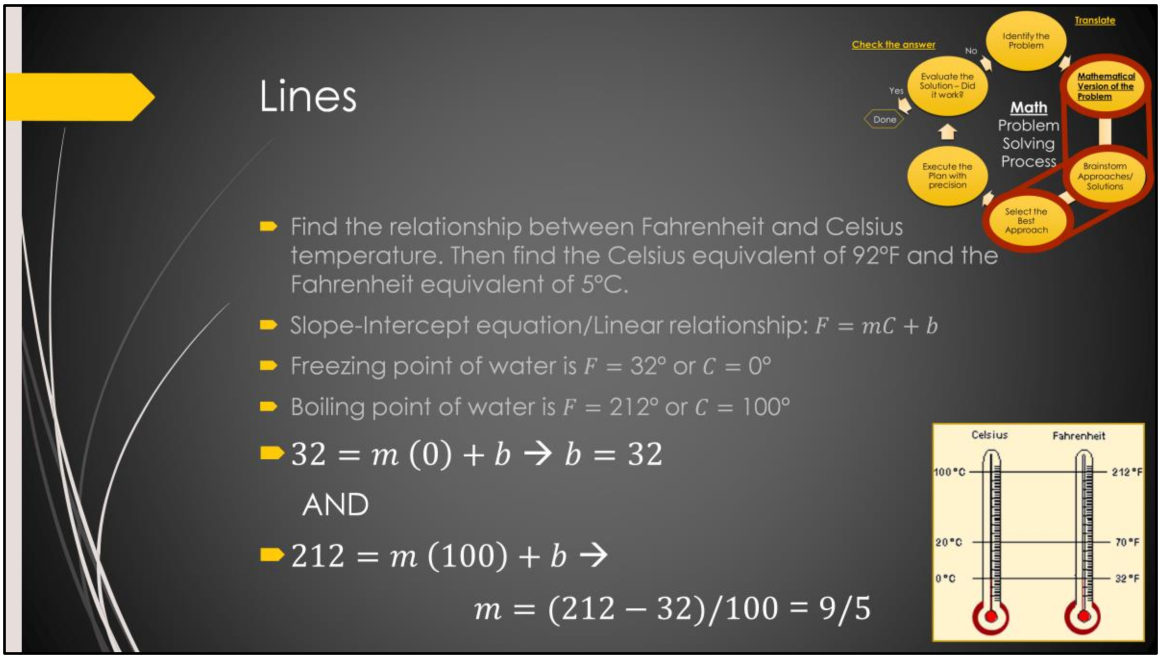


For word problems involving direct relationships, we often use our knowledge of the relationship of lines, in other words, point-slope, or slope-intercept equations, and our knowledge of how to determine the slope of a line.

That is the case with this problem. (Read.)

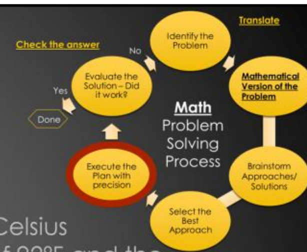


If we plug in F for y and C for x into our slope-intercept equation, and we look up (or know) two sets of points, in this case freezing and boiling points, we can use those to help us answer the question.



If we plug in one set of values into our equation, we can solve for b. Once we have b, we can solve for m.

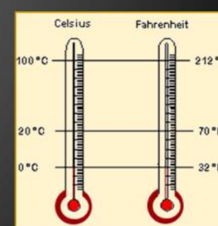
Lines



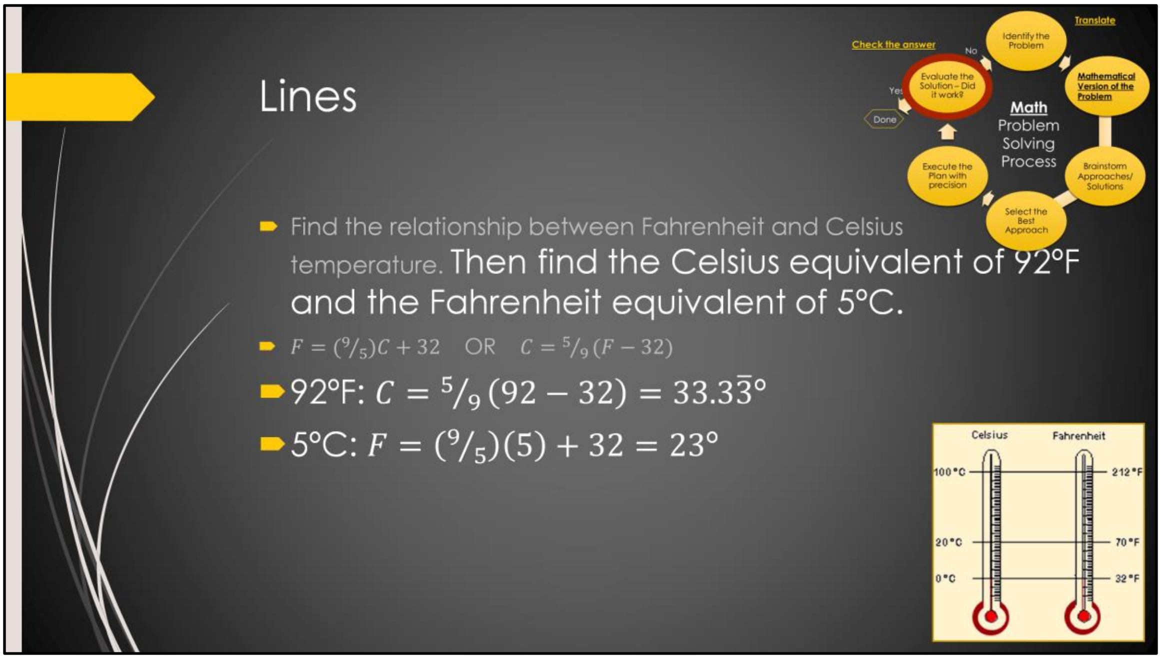
- Find the relationship between Fahrenheit and Celsius temperature. Then find the Celsius equivalent of 92°F and the Fahrenheit equivalent of 5°C.
- Point-Slope equation/Linear relationship:

$$F = mC + b$$

  - $b = 32$ AND $m = 9/5$
- $F = (^9/_5)C + 32$ OR $C = {^5/_9}(F - 32)$

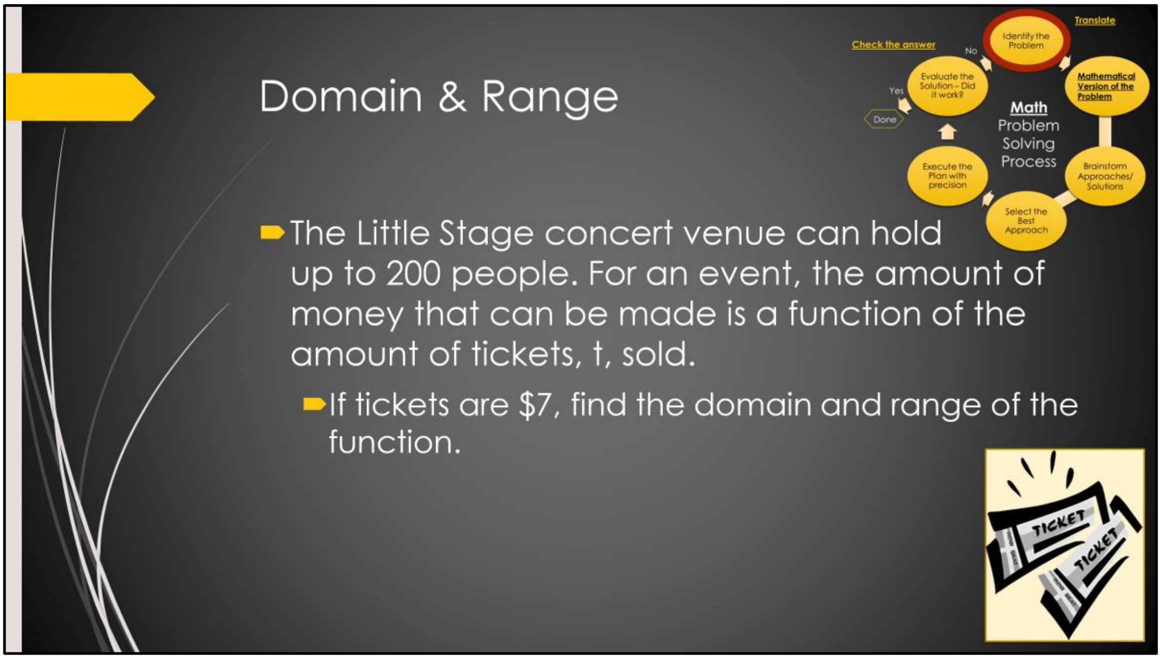


Now that we know both m and b, we can answer the original question by plugging in the know values.
(walk through).

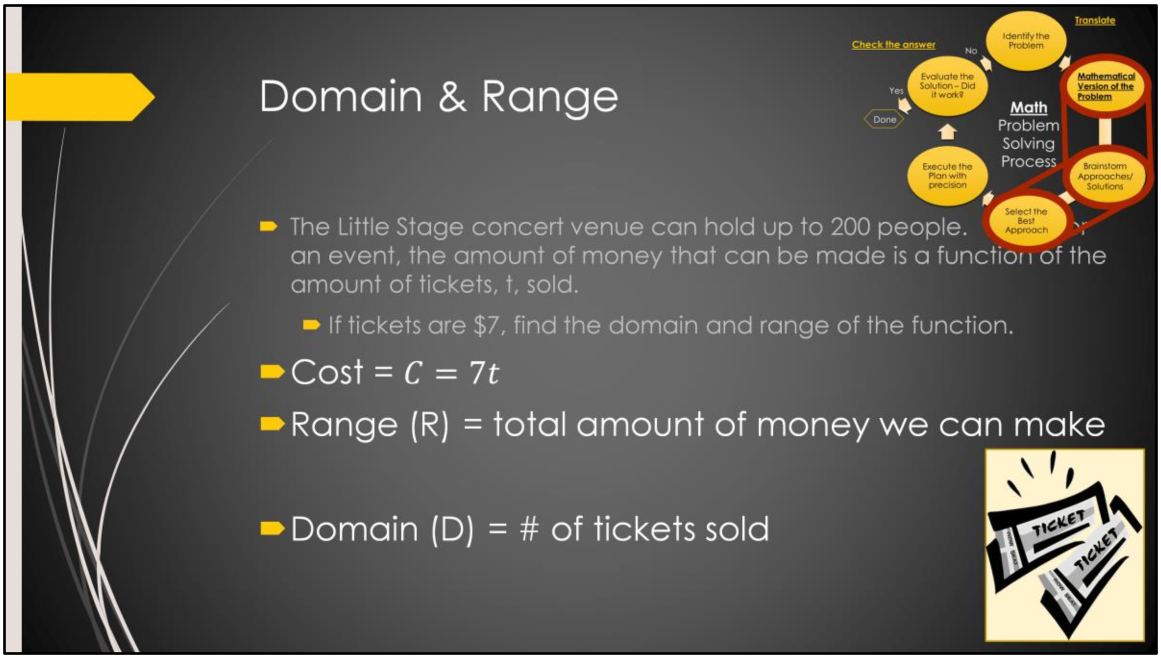


Thus, 92 deg F is 33.33 deg C, and 5 deg C is 23 deg F.

Questions?



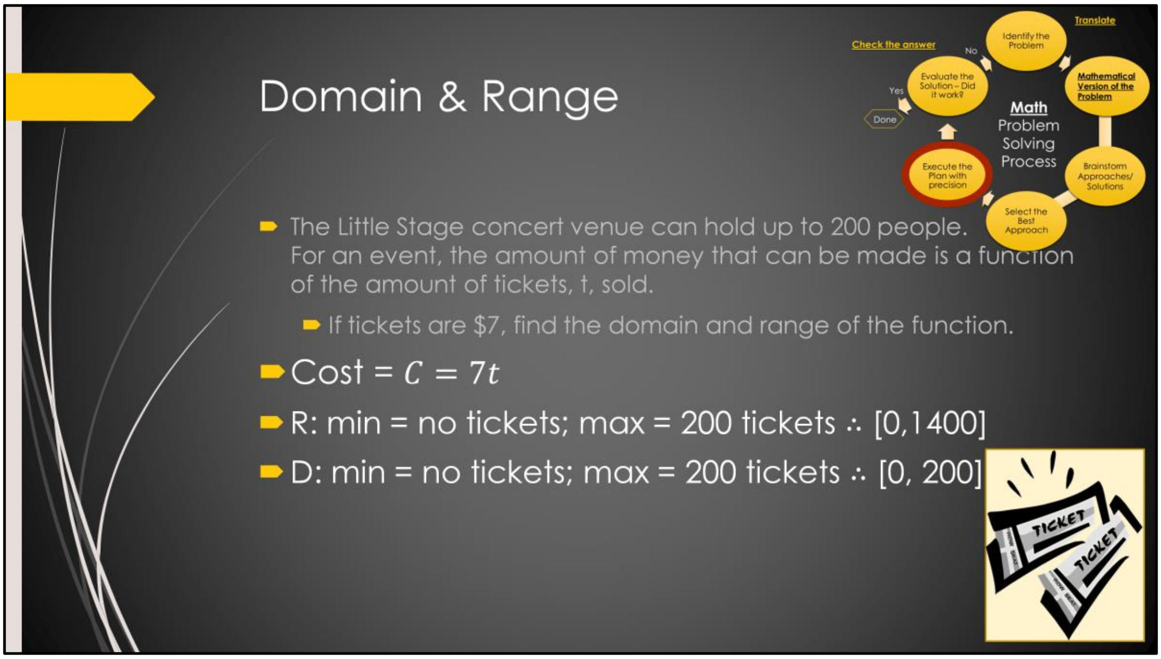
Alright, last topic, finding domains and ranges in word problems.
(read)



Sometimes it is easiest to find range first, and others the domain. You just have to see what information is given to you to determine this.

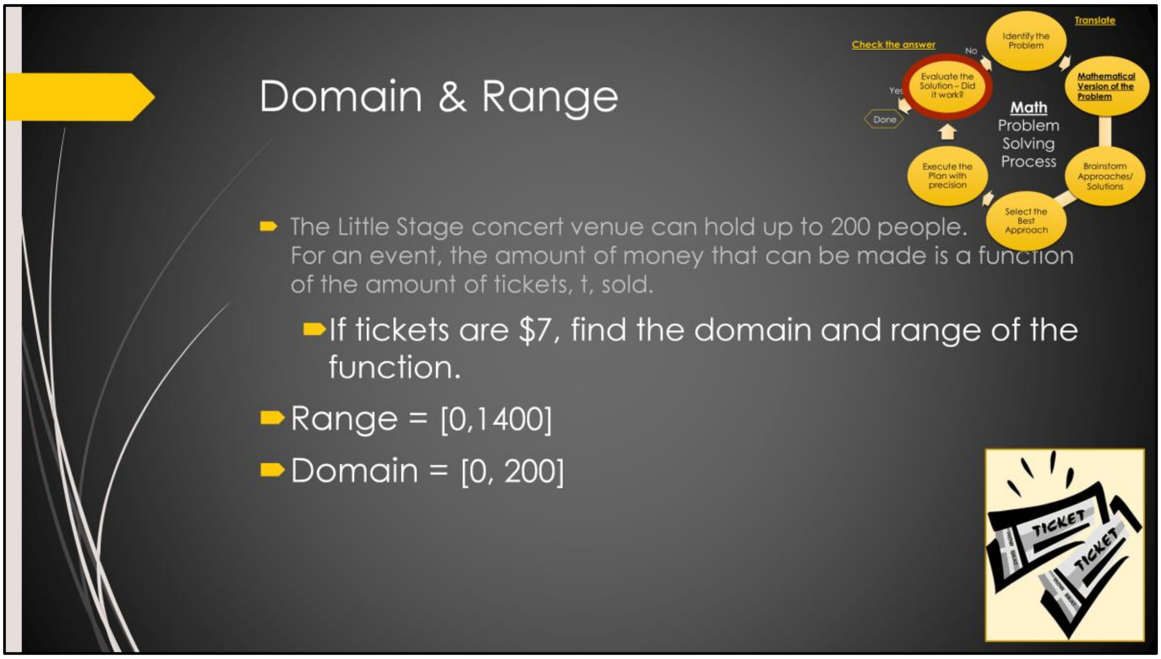
In this case, we can write an equation for Cost, as C = 7t, because each ticket is $7.

That means, the range (dependent variable) of how much money we can make is the total money made, and the domain (independent variable) is the number of tickets sold.



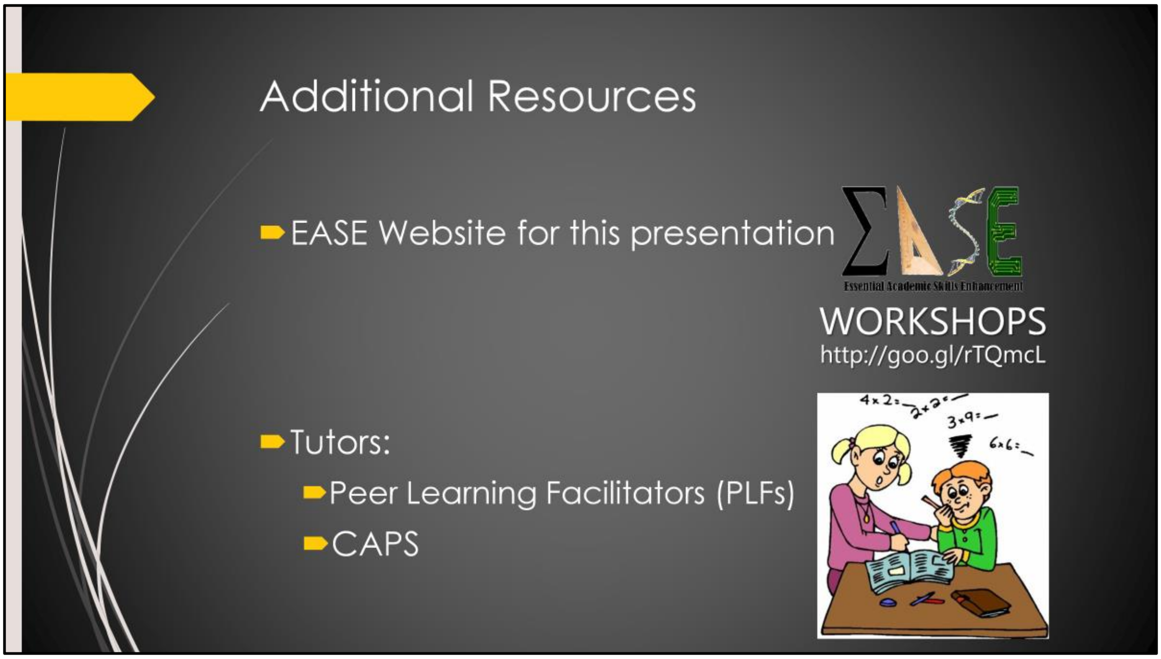
For range, then, we can sell no tickets, and a max of 200, which is stated in the problem. If each ticket is &, that puts the maximum cost of $1400.

For the domain, we can sell no tickets, all the way up to 200.



So the answer is (read).

Any questions?



Again, I want to remind you that this presentation is on the EASE website, with the script that walks you through each problem.

And, tutors are available, not just for your math classes, through ESS and CAPS.

Are there any last minute questions?

I’ll turn it over to you to do the practice problems in just a second, but before you go, we need you to complete our survey. You can do it electronically. When you are finished, call me over to see the confirmation page, and you are free to go. If you’d like to check your answers to the assessment, I have them here, you are free to look over those as well.

Now, take some time to go through your practice problems and call us over for help as needed.

Here is the QR and short URL for the survey you need to complete before you go.