

GRAPH REVIEW

Σ EASE
Essential Academic Skills Enhancement
WORKSHOPS
<http://goo.gl/rTQmcl>

Survey

<http://goo.gl/4AD3OC>
Case sensitive

UNM
STEM GATEWAY

First things first. Welcome to the Math Review workshop, part of the EASE workshop series, organized by the STEM Gateway program and Engineering Student Services.

At the end of the workshop, you'll have to complete a survey for us. If you want to do the survey electronically (which is much appreciated), here are the QR code and short URL. If you'd prefer to do a hard copy version, I'll pass them out at the end of the presentation. To get credit for attending, you'll hand in your assessment sheet, and show me the electronic confirmation, or hand in the paper evaluation, as you leave.

I also want to point out that this Power Point will also be available on the this site through the EASE Workshop series page, so you can always refer back to it at a later time if necessary.

REMEMBER

- Peer Learning Facilitators (PLF's) in select courses/sections are here to help you!
- CAPS has tutors to help you!

- **ALWAYS** check your answer with the original equation!

Before we get too far into the Math, I want to point out two tutoring options available to you, if you feel you need more help than this single workshop refresher. ESS and CAPS have tutors that are FREE and can help you if you need it.

With regards to the math, remember that sometimes, there is no real solution, even if you find one. This can occur when you try to find the square root of a negative number. So, **ALWAYS** check your answer with the original equation.

GRAPHING - LINES

- Slope-intercept equation:

$$y = mx + b$$

- Point-slope equation:

$$y = m(x - x_1) + y_1$$

- Where the slope = $m = \frac{\Delta y}{\Delta x} = (y_2 - y_1)/(x_2 - x_1)$

On to graphing!

The simplest graph is a line, where the slope-intercept equation is $y = mx + b$, where m is the slope and b is the intercept.

This can also be written in point-slope form, with $y = m$ or slope times $(x - x_1) + (y_1)$, where x_1 and y_1 are given coordinates for a point along the line.

And, slope can be represented as the change in y over the change in x , or calculated by $(y_2 - y_1)/(x_2 - x_1)$, where x_1, y_1 is one point on the line, and x_2, y_2 is another.

GRAPHING - LINES

$$y = mx + b$$

$$y = m(x - x_1) + y_1$$

$$m = \frac{\Delta y}{\Delta x} = (y_2 - y_1)/(x_2 - x_1)$$

▪ **To Solve:** Write the point-slope equation for a line that goes through points (-2, -1) and (3, 4).

1. $m = \frac{\Delta y}{\Delta x} = (y_2 - y_1)/(x_2 - x_1) \rightarrow$

$$m = \frac{(4 - (-1))}{(3 - (-2))} = \frac{5}{5} = 1$$


2. $y = m(x - x_1) + y_1 = 1(x - (-2)) + (-1) \rightarrow$

$$y = x + 2 + (-1) = x + 1$$

So, to write the point-slope equation for a line that goes through two given points, (*) we first have to determine the slope using the equation for slope. (*) then, we can use the coordinates and the slope in the point-slope equation, that results in $y = x + 1$.

Are there any questions at this point?

GRAPHING - LINES

▪ Now you try! 

Assessment set 1

Write the equation for a line through
(-2, 1) and (4, -7)

$$y = mx + b$$

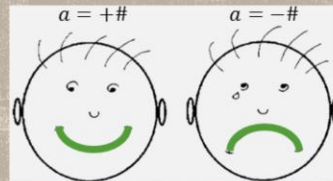
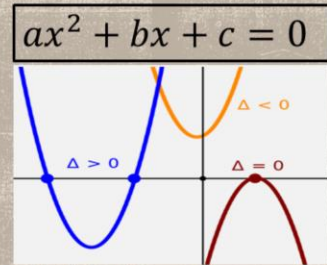
$$y = m(x - x_1) + y_1$$

$$m = \frac{\Delta y}{\Delta x} = (y_2 - y_1)/(x_2 - x_1)$$

Ok, you're turn.

GRAPHING — QUADRATICS

- $\Delta = b^2 - 4ac$ where:
 - The intercept = 2 values of x
 - If $\Delta = -\#$, there is no intercept
 - To find an intercept, set the other coordinate variable to zero.
- Vertex (high or low point) = (h, k)
 - where $h = -b/2a$ and $k = (4ac - b^2)/4a$
- Width and direction of the curve is based on the value of a
 - $|a| < 1$, the graph is fat
 - $|a| > 1$, the graph is skinny



Now, stepping it up a notch, to graph quadratics, it's a bit more complicated. Again, the generic equation is $ax^2 + bx + c = 0$.

(*) First, determine how many intercept points there will be. Use delta to help determine this. $\Delta = b^2 - 4ac$, where if delta is 0, there is 1 intercept point (red on the is graphic), if delta is +, there are 2 (blue curve), and if delta is -, then there are no intercept points (orange).

(*) Next, determine the location of the vertex, that is, the high or low point of the curve. It is located at coordinates (h, k) , where $h = -b/2a$, and $k = (4ac - b^2)/4a$.

(*) Lastly, you can determine the direction and width of the graph based on the value of a. If the absolute value of a is smaller than 1, the curve is fat, and if it is greater than 1, it is skinny. Also, if a is +, then the curve is upward, like a positive face, and if it is negative, it is a negative face, with a downward curve.

Let's put all this together.

$$(h, k): h = -b/2a \text{ and } k = (4ac - b^2)/4a$$

GRAPHING — QUADRATICS

$$ax^2 + bx + c = 0$$

- Find the vertex and intercepts, then graph:

$$y = 3x^2 + x - 2$$

- y intercept: $y = 3(0)^2 + (0) - 2 = 0 + 0 - 2 = -2 \rightarrow$ at $(0, -2)$

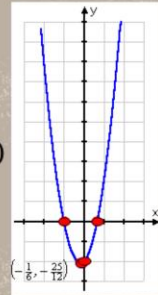
- x intercept: $0 = 3x^2 + x - 2 = (3x - 2)(x + 1) \rightarrow$

$$3x - 2 = 0 \text{ OR } x + 1 = 0$$

$$x = 2/3 \text{ OR } x = -1 \rightarrow \text{at } (-1, 0) \text{ and } (2/3, 0)$$

- Vertex: $h = -1/2(3) \text{ and } k = (4(3)(-2) - (1)^2)/4(3)$

$$h = -1/6 \text{ and } k = -25/12$$



To find the vertex and intercepts for $y = 3x^2 + x - 2$,

(*) we start with the y intercept, where we set $x = 0$, and get the coordinates of the y intercept at $(0, -2)$.

(*) for the x intercept, we do the same where we set $y = 0$. Since x is squared, there are 2 possible solutions, thus 2 intercepts, which is expected if we had solved for delta, as show in the previous slide.

(*) For the vertex, we plug our numbers in to the equations for h and k .


(*) So, to graph this, we know our curve will be upward because a is positive, and we have 4 points we can plot, the y intercept, the two x intercepts, and the vertex. Then we connect the dots.

Are there any questions?

$$(h, k): h = -b/2a \text{ and } k = (4ac - b^2)/4a$$

GRAPHING – QUADRATICS

$$ax^2 + bx + c = 0$$

▪ Now you try! 

Assessment set 1

Find the vertex and intercepts, then graph:

$$y = x^2 - x - 12$$

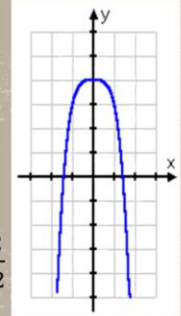
You do the same thing now for this equation.

GRAPHING — DOMAIN & RANGE

- Domain: set of all the starting points, the x values
- Range: the set of all the ending points, the y values

- Solve: determine the domain and range of: $y = \frac{x^2+2-x}{x^2-x-2}$

- Caution! Make sure to NOT divide by 0.
 - Solution: set denominator to 0 to determine what x can NOT be.
 $x^2 - x - 2 = 0 \rightarrow (x - 2)(x + 1) = 0 \rightarrow x = 2 \text{ OR } x = -1$
 Domain = all x not equal to -1 or 2
- To determine range – it is easiest to look at the graph
 Range = all $y \leq 4$



Next, let's talk about domain and range. The domain is the set of all starting points, or possible x values,

(* where the range is the set of all ending points, or possible y values.

(* To determine the domain of this equation, for example, we need to determine which values of x are possible.

(* since this equation is a fraction, we have to be careful to not divide by zero

(* this implies that we can have any value to x except those which result in the denominator = 0, so let's just determine which values those would be and say all but those values. So, $x^2 - x - 2 = 0$, gives us $x = -1$ and 2 , so the domain is all values except for $x = -1$ or 2 .

(* The range is always easiest if we just look at a graph.

(* So, looking at this, what would you say is the range?

(* it is all values of y that are equal or less than 4.

Any questions?

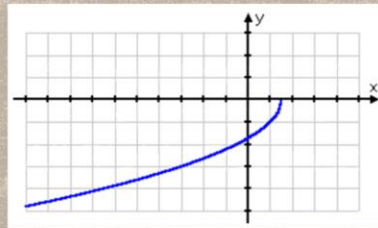
GRAPHING – DOMAIN & RANGE

▪ Now you try! ★

Assessment set 1

Find the domain and range of:

$$y = -\sqrt{-2x + 3}$$



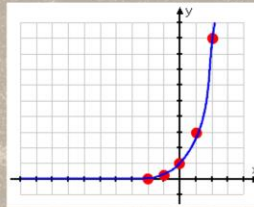
So it's your turn. Find the domain and range of this equation, and here is the graph of it to help you with the range.

GRAPHING - EXPONENTIAL FUNCTIONS

- Evaluate and graph by plugging in coordinate options
 - Some points are too small to graph
 - Some points are too large to graph
 - Only a few “reasonable” points to graph
- Evaluate and graph: $f(x) = 3^x$ at $x = -2, -1, 0, 1, \text{ and } 2$

x	y = 3 ^x
-2	$3^{-2} = 1/9 \approx 0.1$
-1	$3^{-1} = 1/3$
0	$3^0 = 1$
1	$3^1 = 3$
2	$3^2 = 9$

→



Don't forget
about the
asymptote!

To graph exponential functions, it is easiest to just plug in a few values for x , solve for f of x , make a table of those coordinates, and go from there.


But, we have to remember, because of the nature of exponential functions, some points will have a very small number, some will be way too large, and only a few will be just right.

(*) To evaluate and graph $y = 3$ to the x

(*) we create a table with given values of x and plug each in to the equation to get a value of f of x . You can see if we go beyond $x = -2$, or $x = 2$ the numbers would not be manageable. Once we have our points,

(*) we can plot them and connect the dots. Don't forget the asymptotes when doing so, so that the curve has the correct shape.

GRAPHING - EXPONENTIAL FUNCTIONS

▪ Now you try! 

Assessment set 1

Evaluate and graph, at $x = -3, -2, -1, 0, 1, 2,$ and 3 :

$$f(x) = 2^x + 4$$

Don't forget about your asymptotes!

Evaluate and graph.

GRAPHING - TRIGONOMETRY

▪ The starting point is summarized here:

▪ The generic equation:

$$f(t) = A * TrigFunction (Bt - C) + D$$

↕ A = amplitude

▪ large # = higher, fractions = lower, negatives flip the wave.

▪ B = period = $\omega = \langle regular\ period \rangle / |B|$

↔ B increases → period decreases; B decreases → period increases

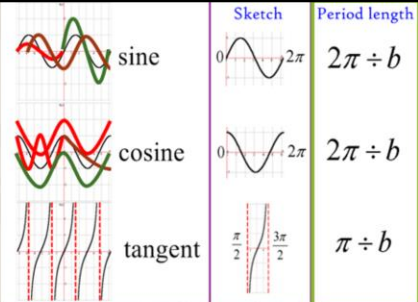
▪ C = phase shift = C/B OR the isolated form of C

▪ Horizontal shift #, in (variable - #)

↔ Watch out! You MUST isolate the variable first.

▪ $\sin(2t - \pi) \rightarrow \sin(2(t - \pi/2))$, phase shift = $\pi/2$

↕ D = vertical shift; moves the entire wave up or down



The last thing we'll go over today is graphing trigonometric functions.

It's best if you remember the starting points, where the basic sin wave intercepts (0, 0), cos is at (0, 1), and tangent have asymptotes, with a single wave looking like these graphs here.

(*) The generic equation is given as the f of t = A times the trig function, so sin, cos, or tan, times (Bt - C) + D.

(*) Where A is the amplitude, that is, the height of the wave. Larger values have higher waves (maroon), smaller values are shallower (red), and a negative will flip the entire wave.

(*) B is the period, calculated by taking the standard period length, summarized here (point to graphic), divided by the absolute value of B. So, if B is larger, the period condenses (red), and if it is smaller, it extends (maroon).

(*) C determines if there is a phase shift, which is a vertical shift in the entire wave, and the magnitude is determined by C/B. But, you have to watch out! When you are trying to determine what the shift is, make sure you isolate the variable first. For example, if we have $\sin(2t - \pi)$, the variable is not isolated, so we need to convert that to $\sin(2(t - \pi/2))$, and the shift is then $\pi/2$.

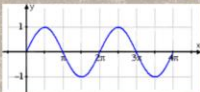
(*) Lastly, D is the vertical shift, which moves the entire wave up and down.

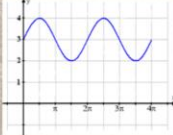
$f(t) = A * \text{TrigFunction} (Bt - C) + D$

GRAPHING - TRIGONOMETRY

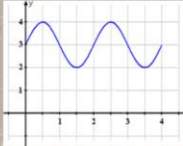
▪ Graph two periods of:

$$g(x) = \sin\left(\pi x + \frac{\pi}{2}\right) + 3$$





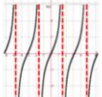

Start: 

Shift vertically: 

Change the period: $\omega = 2\pi/|\pi| = 2$

Shift horizontally: 

Isolate the variable: $\left(\pi x + \frac{\pi}{2}\right) \rightarrow \pi\left(x + \frac{1}{2}\right)$; then C/B = 1/2 to the left

	Sketch	Period length
 sine		$2\pi \div b$
 cosine		$2\pi \div b$
 tangent		$\pi \div b$


Let's walk through an example. I've put these arrows here (see upper equation) to help you remember which value does what to the wave.

If we graph 2 periods, that is 2 full waves, of $g(x)$ for $\sin(\pi x + \pi/2) + 3$

- (*) we start with our standard sin wave. From there
- (*) we shift it vertically by 3 because of the D value of 3
- (*) then we change the period as a result of $b = \pi$, the easiest thing to do is just change the axis values, so from $\pi, 2\pi$, etc, to 1, 2, etc.
- (*) Lastly, we have to shift the graph horizontally, but remember, you have to isolate the variable first before you can determine how far the shift is. And, since we have a + sign for C, the shift is to the left.

Any questions?

GRAPHING - TRIGONOMETRY


▪ Now you try! 

Assessment set 1

Graph two periods of:

$$y = 3 \sin(2x - \pi) - 1$$

$$f(t) = A * TrigFunction (Bt - C) + D$$


period/B C/B

Last one! It's your turn to graph 2 periods of $y = 3 \sin(2x - \pi)$.

ADDITIONAL RESOURCES

- Presentation on EASE website (<http://goo.gl/rTQmCL>)
- Peer Learning Facilitators (PLF's) in select courses/sections are here to help you!
- CAPS Tutors
- www.purplemath.com
- <https://www.desmos.com/calculator> (online graphing)

Before we wrap up, I want to mention again, this presentation, including the script, is available on the EASE website. So, you can go back and refresh your memory on this at any time.

Also, ESS and CAPS have tutors to help you

And, purplemath.com (which many of these examples came from) is great for more examples, and desmos.com allows you to do online graphing, so you can always check yourself.

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Are there any last minute questions?

There is a second set of assessment questions if you want to stick around and get more practice. We will go around and help you as necessary.

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